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If you require any further information and assistance feel free to contact us on academicascs@gmail.com.

Tutorial Sheet No 6

1

a i) $\int_1^3 k(4-x)\,dx = 1$

$k\left[4x - \frac{x^2}{2}\right]_1^3 = 1$

$k\left\{\left(12 - \frac{9}{2}\right) - \left(4 - \frac{1}{2}\right)\right\} = 1$

$k\left\{\frac{15}{2} - \frac{7}{2}\right\} = 1$

$k\left\{\frac{8}{2}\right\} = 1 \Rightarrow 4k = 1 \Rightarrow k = 1/4$

$\therefore f(x) = \frac{(4-x)}{4}$

ii) $Pr(1.2 < X < 2.4)$

$= \int_{1.2}^{2.4} \frac{(4-x)}{4}\,dx$

$= \frac{1}{4}\left[4x - \frac{x^2}{2}\right]_{1.2}^{2.4}$

$= \frac{1}{4}\left\{\left(4\times 2.4 - \frac{2.4^2}{2}\right) - \left(4\times 1.2 - \frac{1.2^2}{2}\right)\right\}$

$= \frac{1}{4}\left\{(9.6 - 2.88) - (4.8 - 0.72)\right\}$

$= \frac{1}{4}\left\{6.72 - 4.08\right\}$

$= \frac{2.64}{4}$ i.e $Pr(1.2 < X < 2.4) = 0.66$

iii) $E(X) = \int_1^3 \frac{x(4-x)}{4}\,dx$

$= \frac{1}{4}\int_1^3 (4x - x^2)\,dx$

$$= \frac{1}{4}\left[\frac{\cancel{4}^{2}x^2}{\cancel{2}} - \frac{x^3}{3}\right]_1^3$$

$$= \frac{1}{4}\left\{\left(2\cdot 3^2 - \frac{3^3}{3}\right) - \left(2\cdot 1^2 - \frac{1^3}{3}\right)\right\}$$

$$= \frac{1}{4}\left\{(18 - 9) - \left(2 - \frac{1}{3}\right)\right\}$$

$$= \frac{1}{4}\left\{9 - \frac{5}{3}\right\}$$

$$= \frac{1}{4}\left\{\frac{27-5}{3}\right\}$$

$$= \frac{1}{4}\cdot\frac{22}{3} \quad \text{i.e} \quad E(X) = \frac{11}{6}$$

$$E(X^2) = \int_1^3 \frac{(4-x)}{4} x^2\,dx$$

$$= \frac{1}{4}\int_1^3 (4x^2 - x^3)\,dx$$

$$= \frac{1}{4}\left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_1^3$$

$$= \frac{1}{4}\left\{\left(\frac{4\cdot 3^3}{3} - \frac{3^4}{4}\right) - \left(\frac{4\cdot 1^3}{3} - \frac{1^4}{4}\right)\right\}$$

$$= \frac{1}{4}\left\{\left(36 - \frac{81}{4}\right) - \left(\frac{4}{3} - \frac{1}{4}\right)\right\}$$

$$= \frac{1}{4}\left\{\left(\frac{144-81}{4}\right) - \left(\frac{16-3}{12}\right)\right\}$$

$$= \frac{1}{4}\left\{\frac{63}{4} - \frac{13}{12}\right\}$$

$$= \frac{1}{4}\left\{\frac{189-13}{12}\right\}$$

$$= \frac{176}{4 \times 12}$$

$$= \frac{11}{3} \quad i.e \quad E(X^2) = \frac{11}{3}$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \frac{11}{3} - \left(\frac{11}{6}\right)^2$$

$$= \frac{11}{3} - \frac{121}{36}$$

$$= \frac{132 - 121}{36}$$

$$= \frac{11}{36} \quad i.e \quad Var(X) = \frac{11}{36}$$

Standard Derivation $= \sqrt{\frac{11}{36}}$

$$= 0.5527708$$

b) $\because \int_1^4 c(x+2)\,dx = 1$

$$c\int_1^4 (x+2)\,dx = 1$$

?

$$c\left[\frac{x^2}{2} + 2x + \cancel{c}\right]_1^4 = 1$$

$$c\left\{\left(\frac{4^2}{2} + 2.4\right) - \left(\frac{1}{2} + 2.1\right)\right\} = 1$$

$$c\left\{(8+8) - \left(\frac{1}{2} + 2\right)\right\} = 1$$

$$c\left\{16 - \frac{5}{2}\right\} = 1$$

$$c\left\{\frac{32-5}{2}\right\} = 1$$

$$c\,\frac{27}{2} = 1 \Rightarrow c = \frac{2}{27}$$

$$\therefore f(x) = \frac{2}{27}(x+2)$$

ii) $$E(X) = \int_1^4 \frac{2}{27}(x+2)x\,dx$$

$$= \frac{2}{27}\int_1^4 (x^2+2x)\,dx$$

$$= \frac{2}{27}\left[\frac{x^3}{3} + x^2 + k\right]_1^4$$

$$= \frac{2}{27}\left\{\left(\frac{4^3}{3}+4^2+k\right) - \left(\frac{1^3}{3}+1^2+k\right)\right\}$$

$$= \frac{2}{27}\left\{\left(\frac{64}{3}+16\right) - \left(\frac{1}{3}+1\right)\right\}$$

$$= \frac{2}{27}\left\{\frac{112}{3} - \frac{4}{3}\right\}$$

$$= \frac{2}{\cancel{27}}\left(\frac{\cancel{108}^{4}}{3}\right)$$

$$= \frac{8}{3} \quad \text{i.e } E(X) = \frac{8}{3}$$

$$E(X^2) = \int_1^4 \frac{2}{27}(x+2)x^2\,dx$$

$$= \frac{2}{27}\int_1^4 (x^3+2x^2)\,dx$$

$$= \frac{2}{27}\left[\frac{x^4}{4} + \frac{2x^3}{3} + k\right]_1^4$$

$$= \frac{2}{27}\left\{\left(\frac{4^4}{4} + \frac{2.4^3}{3}\right) - \left(\frac{1^4}{4} + \frac{2}{3}.1^3\right)\right\}$$

$$= \frac{2}{27}\left\{\left(64 + \frac{128}{3}\right) - \left(\frac{1}{4} + \frac{2}{3}\right)\right\}$$

$$= \frac{2}{27}\left\{\left(\frac{192+128}{3}\right) - \left(\frac{3+8}{12}\right)\right\}$$

$$= \frac{2}{27}\left\{\frac{320}{3} - \frac{11}{12}\right\}$$

$$= \frac{2}{27}\left\{\frac{1280 - 11}{12}\right\}$$

$$= \frac{\cancel{2}}{27}\left\{\frac{\cancel{1269}^{423}}{\cancel{12}_{\cancel{4}_{2}}}\right\}$$

$$= \frac{\cancel{423}^{47}}{3\cancel{27}.2} \qquad \text{i.e } E(X^2) = \frac{47}{6}$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \frac{47}{6} - \left(\frac{8}{3}\right)^2$$

$$= \frac{47}{6} - \frac{64}{9}$$

$$= \frac{141 - 128}{18}$$

$$= \frac{13}{18}$$

iii) $$\int_1^a f(x)\,dx = \frac{1}{2}$$

$$\int_1^a \frac{2}{27}(x+2)\,dx = \frac{1}{2}$$

$$\frac{2}{27}\int_1^a (x+2)\,dx = \frac{1}{2}$$

$$\frac{2}{27}\left[\frac{x^2}{2} + 2x + k\right]_1^a = \frac{1}{2}$$

$$\frac{2}{27}\left\{\left(\frac{a^2}{2} + 2a\right) - \left(\frac{1}{2} + 2\right)\right\} = \frac{1}{2}$$

$$\left\{\left(\frac{a^2}{2} + 2a\right) - \frac{5}{2}\right\} = \frac{1}{2} \times \frac{27}{2}$$

$$\frac{a^2}{2} + 2a - \frac{5}{2} = \frac{27}{4}$$

$$\frac{a^2}{2} + 2a = \frac{27}{4} + \frac{5}{2}$$

$$\frac{a^2}{2} + 2a = \frac{37}{4}$$

$$2a^2 + 8a = 37$$

$$2a^2 + 8a - 37 = 0$$

$$a = \frac{-8 \pm \sqrt{8^2 - 4.2.-37}}{2.2}$$

$$= \frac{-8 \pm \sqrt{64 + 296}}{4}$$

$$= \frac{-8 \pm \sqrt{360}}{4}$$

$$= \frac{-8 \pm 6\sqrt{10}}{4}$$

$$= \frac{-4 \pm 3\sqrt{10}}{2}$$

$$= \frac{-13.48683298}{2}, \frac{5.486832981}{2}$$

$$= -6.74341649, 2.743416491$$

$$\therefore a = 2.7434$$

---

2 a) $Pr(X < 40.1)$

$$= Pr\left(\frac{X - 62.5}{12.4} < \frac{40.1 - 62.5}{12.4}\right)$$

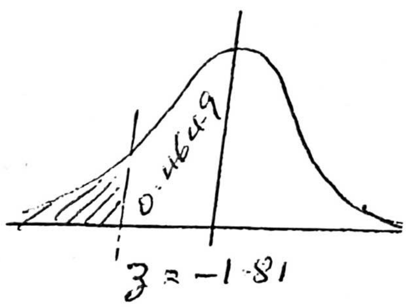
$$= Pr\left(Z < \frac{-22.4}{12.4}\right)$$

$$= Pr(Z < -1.806451613)$$

$$= Pr(Z < -1.81)$$

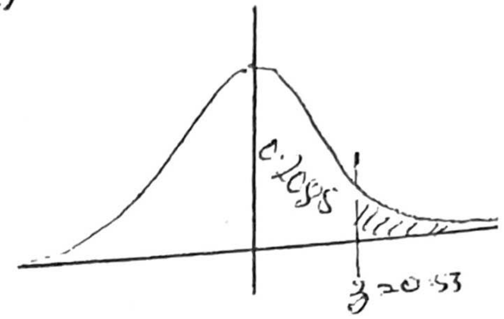
$$= 0.5 - 0.4649)$$

$$= 0.0351$$



b) $Pr(X > 69.3)$

$= Pr\left(\frac{X-62.5}{12.4} > \frac{69.3-62.5}{12.4}\right)$



$= Pr\left(Z > \frac{6.8}{12.4}\right)$

$= Pr(Z > 0.548387096)$

$= Pr(Z > 0.55)$

$= 0.5 - 0.2088$

$= 0.2912$

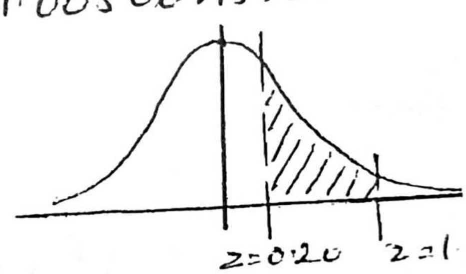
c) $Pr(65.0 < X < 75.0)$

$= Pr\left(\frac{65.0-62.5}{12.4} < \frac{X-62.5}{12.4} < \frac{75.0-62.5}{12.4}\right)$

$= Pr\left(\frac{2.5}{12.4} < Z < \frac{12.5}{12.4}\right)$

$= Pr(0.201612903 < Z < 1.008064516)$

$= Pr(0.20 < Z < 1.01)$



$= 0.3438 - 0.0793$

$= 0.2645$

d) $Pr(57.4 < X < 67.6)$

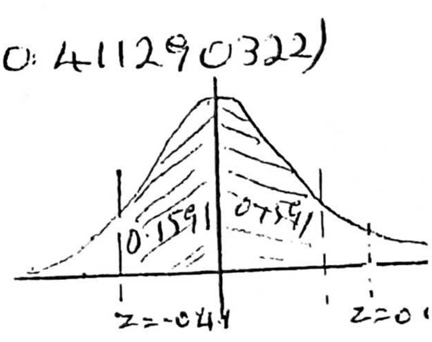
$= Pr\left(\frac{57.4-62.5}{12.4} < \frac{X-62.5}{12.4} < \frac{67.6-62.5}{12.4}\right)$

$= Pr\left(\frac{-5.1}{12.4} < Z < \frac{5.1}{12.4}\right)$

$= Pr(-0.411290322 < Z < 0.411290322)$

$= Pr(-0.41 < Z < 0.41)$

$= 2Pr(0 < Z < 0.41)$



$= 2 \times 0.1591$

$= 0.3182$

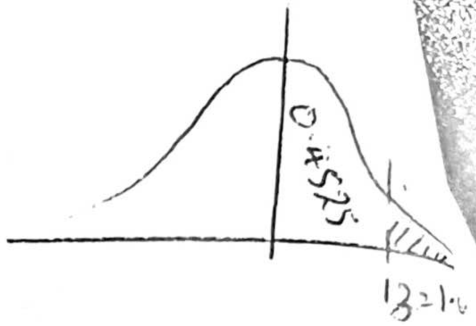
---

3 P(X will have a positive value)

$= Pr(X > 0)$

$= Pr\left(\frac{X-\{-5\}}{3} > \frac{0-\{-5\}}{3}\right)$

$= Pr\left(Z > \frac{5}{3}\right)$



$= Pr(Z > 1.6666667)$

$= Pr(Z > 1.67)$

$= 0.5 - 0.4525$

$= 0.0475$ i.e $Pr(X > 0) = 0.0475$.

b) The situation discussed here represents a binomial distribution

Then let $p = Pr(X > 0)$ i.e $p = 0.0475$

and $1-p = 1-0.0475$ or $1-p = 0.9525$

Let Y represents the number of items with a positive value for the characteristic

(Binomial because we know the probability of occurance and probability of not occurring)

$\therefore Pr(Y = 4)$

$= {}^{10}C_4 (0.0475)^4 (0.9525)^6$

$= \frac{10!}{4!\,6!} (0.0475)^4 (0.9525)^6$

$= \frac{10.9.8.7}{1.2.3.4} (0.0475)^4 (0.9525)^6$

$= 210 (0.0475)^4 (0.9525)^6$

$= 210 \times 5.090664062 \times 10^{-6} \times 0.7467752$

$= 7.983321867 \times 10^{-4}$

$= 0.0007983321867$

$= 0.0008$

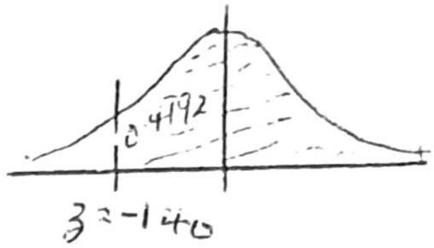
---

4 Let X be the mass of a cabbage

a) $Pr(X > 0.79)$

$= Pr\left(\frac{X-1}{0.15} > \frac{0.79-1}{0.15}\right)$

$= Pr\left(Z > \frac{-0.21}{0.15}\right)$



$= Pr(Z > -1.40)$

$= 0.5 + 0.4192$

$= 0.9192$ i.e $Pr(X > 0.79) = 0.9192$

Estimated number of cabbages with mass greater than $\overset{0.79}{1}$ kg $= 0.9192 \times 800$

$= 735.36 \simeq 735$

b) $Pr(X < 1.13)$

$= Pr\left(\frac{X-1}{0.15} < \frac{1.13-1}{0.15}\right)$

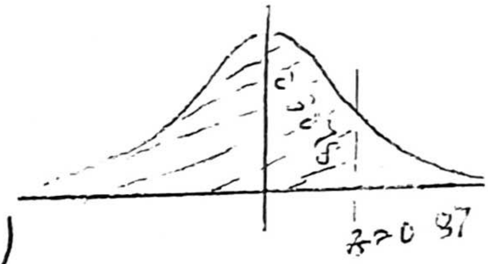
$= Pr\left(Z < \frac{0.13}{0.15}\right)$

$= Pr(Z < 0.8666\dot{6})$

$= Pr(Z < 0.87)$

$= 0.3078 + 0.5$

$= 0.8078$



Estimated number of cabbages with mass less than 1.13kg $= 800 \times 0.8078$

$= 646.24 \simeq 646$

c) $Pr(0.85 < X < 1.15)$

$= Pr\left(\frac{0.85-1}{0.15} < \frac{X-1}{0.15} < \frac{1.15-1}{0.15}\right)$

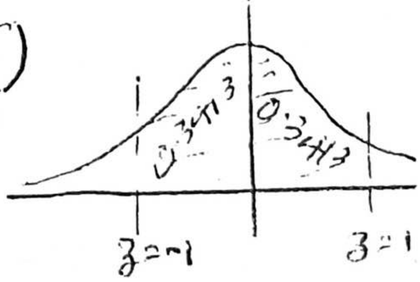
$= Pr\left(\frac{-0.15}{0.15} < Z < \frac{0.15}{0.15}\right)$

$= Pr(-1 < Z < 1)$

$= 2Pr(0 < Z < 1)$

$= 2 \times 0.3413$

$= 0.6826$



Estimated number of cabbages with

mass between 0.85kg and 1.15kg is 800×0.6826
$546.08 \simeq 546$

d) $Pr(0.75 < X < 1.29)$

$= Pr\left(\frac{0.75-1}{0.15} < \frac{X-1}{0.15} < \frac{1.29-1}{0.15}\right)$

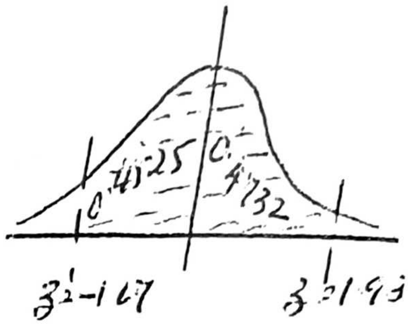
$= Pr\left(\frac{-0.25}{0.15} < Z < \frac{0.29}{0.15}\right)$

$= Pr(-1.666\dot{6} < Z < 1.933\dot{3})$

$= Pr(-1.67 < Z < 1.93)$

$= 0.4525 + 0.4732$

$= 0.9257$



Estimated number of cabbages with a mass between 0.75kg and 1.29kg is
$800 \times 0.9257 = 740.56 \simeq 741$.

5

$Pr(X > 65.6) = 0.0212$

$Pr\left(\frac{X-52.5}{\sigma} > \frac{65.6-52.5}{\sigma}\right) = 0.0212$

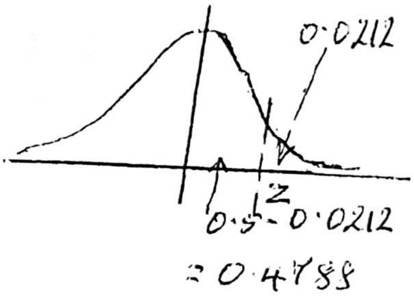
$Pr\left(Z > \frac{13.1}{\sigma}\right) = 0.0212$

from tables $Z = 2.03$

$\therefore 2.03 > \frac{13.1}{\sigma}$

$\sigma > \frac{13.1}{2.03}$

$> 6.45320197 \Rightarrow \sigma \simeq 6.50$



i) $Pr(X > 56.7)$

$= Pr\left(\frac{X-52.5}{6.50} > \frac{56.7-52.5}{6.50}\right)$

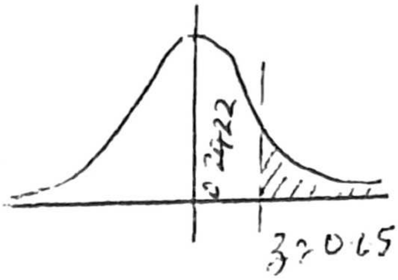
$= Pr\left(Z > \frac{4.2}{6.50}\right)$

$= Pr(Z > 0.646153846)$

$= Pr(Z > 0.65)$

$= 0.5 - 0.2422$

$= 0.2578$



i.e $Pr(X > 56.7) = 0.2578$

ii) $Pr(X < 49.8)$

$= Pr\left(\frac{X - 52.5}{6.50} < \frac{49.8 - 52.5}{6.50}\right)$

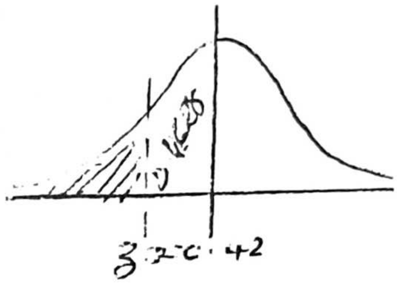
$= Pr\left(Z < \frac{-2.7}{6.50}\right)$

$= Pr(Z < -0.415384615)$

$= Pr(Z < -0.42)$

$= 0.5 - 0.1628$

$= 0.3372$



$\therefore Pr(X < 49.8) = 0.3372$

iii) $Pr(59.6 < X < 68.9)$

$= Pr\left(\frac{59.6 - 52.5}{6.50} < \frac{X - 52.5}{6.50} < \frac{68.9 - 52.5}{6.50}\right)$

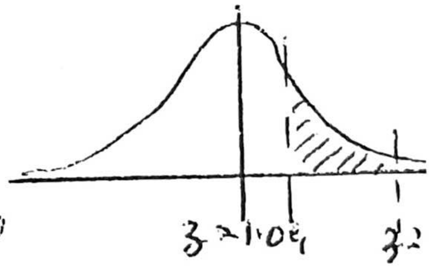
$= Pr\left(\frac{7.1}{6.50} < Z < \frac{16.4}{6.50}\right)$

$= Pr(1.092307692 < Z < 2.523076923)$

$= Pr(1.09 < Z < 2.52)$

$= 0.4941 - 0.2621$

$= 0.1320$



i.e $Pr(59.6 < X < 68.9) = 0.1320$

iv) $Pr(42.4 < X < 62.9)$

$= Pr\left(\frac{42.4 - 52.5}{6.50} < \frac{X - 52.5}{6.50} < \frac{62.9 - 52.5}{6.50}\right)$

$= Pr\left(\frac{-10.1}{6.50} < Z < \frac{10.4}{6.50}\right)$

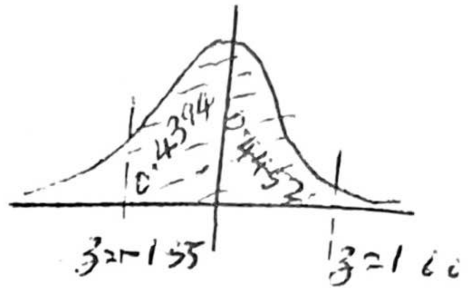
$= Pr(-1.553846154 < Z < 1.60)$

$= Pr(-1.55 < Z < 1.60)$

$= 0.4394 + 0.4452$

$= 0.8846$

$\therefore Pr(42.4 < X < 62.4) = 0.8846$



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6 The situation in this problem follows a Binomial Distribution, it can be approximately defined as Normal Distribution

Mean $= np$

$= 1000 \times 1/10$

$= 100$ i.e. Mean $= 100$

Variance $= 1000 \times \frac{1}{10} \times \frac{9}{10}$

$= 90$ i.e. Variance $= 90$

Let X be the number of mis shapes in the sample.

a) $Pr(X < 80)$

$\simeq Pr(X < 79.5)$

$= Pr\left(\frac{X - 100}{\sqrt{90}} < \frac{79.5 - 100}{\sqrt{90}}\right)$

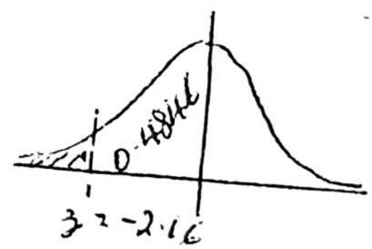
$= Pr\left(Z < \frac{-20.5}{\sqrt{90}}\right)$

$= Pr(Z < -2.160889734)$

$= Pr(Z < -2.16)$

$= 0.5 - 0.4846$

$= 0.0154$ i.e. $Pr(X < 80) \simeq 0.0154$



b) $Pr(90 \leq X \leq 115)$

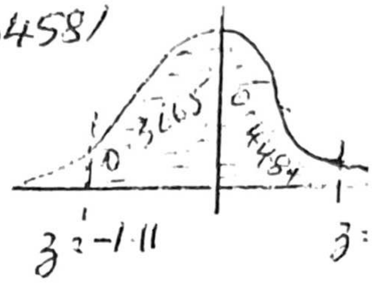
$\simeq Pr(89.5 < X < 115.5)$

$= Pr\left(\frac{89.5 - 100}{} < \frac{X - 100}{} < \frac{115.5 - 100}{}\right)$

$= Pr\left(\frac{-10.5}{\sqrt{90}} < Z < \frac{15.5}{\sqrt{90}}\right)$

$= Pr(-1.106797181 < Z < 1.633843458)$

$= Pr(-1.11 < Z < 1.63)$



$= 0.3665 + 0.4484$

$= 0.8149$

i.e $Pr(90 \le X \le 115) \simeq 0.8149$

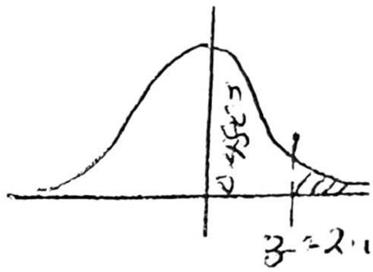
c) $Pr(X \ge 120)$

$\simeq Pr(X > 119.5)$

$= Pr\left(\frac{X-100}{\sqrt{90}} > \frac{119.5-100}{\sqrt{90}}\right)$

$= Pr\left(Z > \frac{19.5}{\sqrt{90}}\right)$

$= Pr(Z > 2.055480479)$



$= Pr(Z > 2.06)$

$= 0.5 - 0.4803$

$= 0.0197$

i.e $Pr(X \ge 120) \simeq 0.0197$

---

4 Mean $= 0.70 \times 100$

$= 70$

Variance $= 100 \times 0.7 \times 0.3$

$= 21$

Let X be the number of mountain bike sold

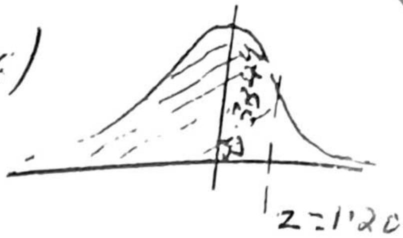
a) $Pr(X \le 75)$

$\simeq Pr(X < 75.5)$

$= Pr\left(\frac{X-70}{\sqrt{21}} < \frac{75.5-70}{\sqrt{21}}\right)$

$= Pr\left(Z < \frac{5.5}{\sqrt{21}}\right)$

$= Pr(Z < 1.200198396)$



$= Pr(Z < 1.20)$

$= 0.5 + 0.3849$

$= 0.8849$

i.e $Pr(X \leq 75) = 0.8849$

b) $Pr(60 \leq X \leq 75)$

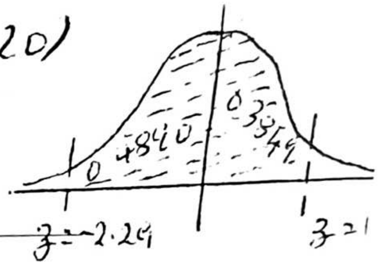
$\simeq Pr(59.5 < X < 75.5)$

$= Pr\left(\dfrac{59.5 - 70}{\sqrt{21}} < \dfrac{X - 70}{\sqrt{21}} < \dfrac{75.5 - 70}{\sqrt{21}}\right)$

$= Pr\left(\dfrac{-10.5}{\sqrt{21}} < Z < \dfrac{5.5}{\sqrt{21}}\right)$

$= Pr(-2.291287847 < Z < 1.20)$

$= Pr(-2.29 < Z < 1.20)$



$= 0.4840 + 0.3849$

$= 0.8739$

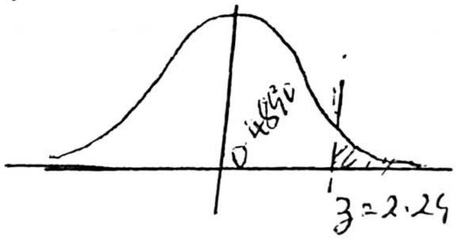
i.e $Pr(60 \leq X \leq 75) \simeq 0.8739$

c) $Pr(X > 80)$

$\simeq Pr(X > 80.5)$

$= Pr\left(\dfrac{X - 70}{\sqrt{21}} > \dfrac{80.5 - 70}{\sqrt{21}}\right)$

$= Pr\left(Z > \dfrac{10.5}{\sqrt{21}}\right)$



$= Pr(Z > 2.29)$

$= 0.5 - 0.4890$

$= 0.0110$ i.e $Pr(X > 80) \simeq 0.0110$

---

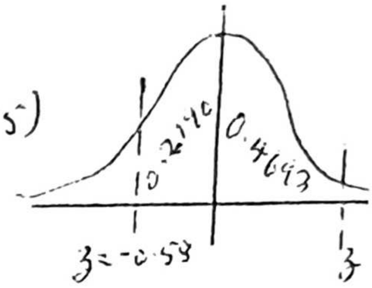
8 X is the number of bacteria on a plate. This follows a Poisson distribution that can approximated by normal distribution i.e $X \sim N(60, 60)$

$Pr(55 < X < 75)$

$\simeq Pr(55.5 < X < 74.5)$

$= Pr\left(\frac{55.5-60}{\sqrt{60}} < \frac{X-60}{\sqrt{60}} < \frac{74.5-60}{\sqrt{60}}\right)$

$= Pr\left(\frac{-4.5}{\sqrt{60}} < Z < \frac{14.5}{\sqrt{60}}\right)$



$= Pr(-0.58094750 < Z < 1.87194195)$

$= Pr(-0.58 < Z < 1.87)$

$= 0.2190 + 0.4693$

$= 0.6883$

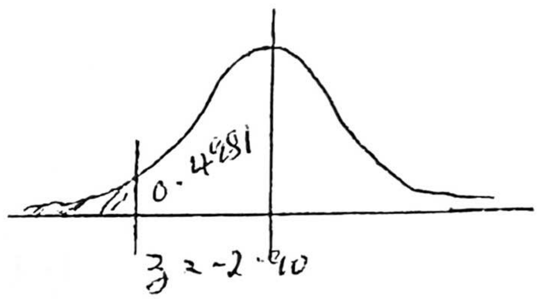
ie $Pr(55 < X < 75) \simeq 0.6883$

$Pr(X < 38)$

$\simeq Pr(X < 37.5)$

$= Pr\left(\frac{X-60}{\sqrt{60}} < \frac{37.5-60}{\sqrt{60}}\right)$

$= Pr\left(Z < \frac{-22.5}{\sqrt{60}}\right)$



$= Pr(Z < -2.9047375)$

$= Pr(Z < -2.90)$

$= 0.5 - 0.4981$

$= 0.0019$ ie $Pr(X < 38) \simeq 0.0019$

No. of plates that are rejected

$= 2000\ Pr(X < 38)$

$= 2000 \times 0.0019$

$= 3.8$

$\simeq 4$

---

9 X is the number of calls received by a telephone system per hour. This follows a Poisson distribution with mean 30 and can be

approximated by normal distribution, i.e. $X \sim N($

a) $Pr(X > 33)$

$\simeq Pr(X > 33.5)$

$= Pr\left(\frac{X-30}{\sqrt{30}} > \frac{33.5-30}{\sqrt{30}}\right)$

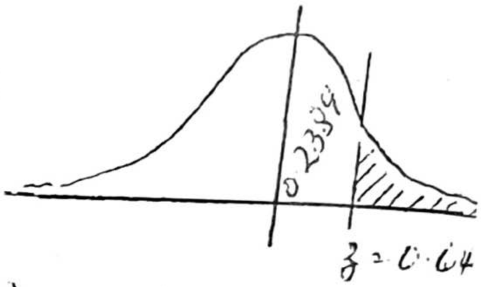
$= Pr\left(Z > \frac{3.5}{\sqrt{30}}\right)$

$= Pr(Z > 0.63900965)$

$= Pr(Z > 0.64)$

$= 0.5 - 0.2389$

$= 0.2611$ i.e. $Pr(X > 33) \simeq 0.2611$



b) $Pr(25 \leq X \leq 28)$

$\simeq Pr(24.5 < X < 28.5)$

$= Pr\left(\frac{24.5-30}{\sqrt{30}} < \frac{X-30}{\sqrt{30}} < \frac{28.5-30}{\sqrt{30}}\right)$

$= Pr\left(\frac{-5.5}{\sqrt{30}} < Z < \frac{-1.5}{\sqrt{30}}\right)$

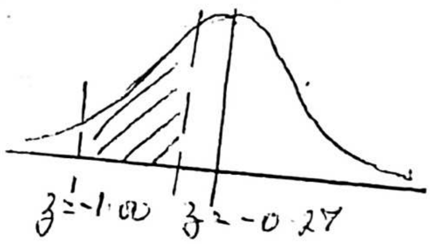
$= Pr(-1.004158022 < Z < -0.273861278)$

$= Pr(-1.00 < Z < -0.27)$

$= 0.3413 - 0.1064$

$= 0.2349$

i.e. $Pr(25 \leq X \leq 28) \simeq 0.2349$



c) $Pr(X = 34)$

$\simeq Pr(33.5 < X < 34.5)$

$= Pr\left(\frac{33.5-30}{\sqrt{30}} < \frac{X-30}{\sqrt{30}} < \frac{34.5-30}{\sqrt{30}}\right)$

$= Pr\left(\frac{3.5}{\sqrt{30}} < Z < \frac{4.5}{\sqrt{30}}\right)$

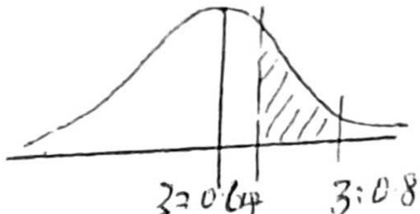
$= Pr(0.63900965 < Z < 0.821583836)$

$= Pr(0.64 < Z < 0.82)$

$= 0.2939 - 0.2389$

$= 0.0550$

$\therefore \Pr(X=34) \simeq 0.0550$



10 The probability density function for exponential distribution is $f(t) = \lambda e^{-\lambda t}$

$E(t)$ = the mean

$$= \int_0^\infty t f(t)\, dt$$

$$= \int_0^\infty t \lambda e^{-\lambda t}\, dt$$

$$= \lambda \int_0^\infty t e^{-\lambda t}\, dt \qquad u = t \quad du/dt = 1$$

$$dv = e^{-\lambda t} dt$$

$$= \lambda \left\{ -t \frac{e^{-\lambda t}}{\lambda} \Big|_0^\infty + \frac{1}{\lambda} \int_0^\infty e^{-\lambda t}\, dt \right\} \qquad v = -e^{-\lambda t}/\lambda$$

$$= \int_0^\infty e^{-\lambda t}\, dt$$

$$= \left[-\frac{e^{-\lambda t}}{\lambda} + k \right]_0^\infty = \frac{1}{\lambda}$$

Now the mean $= 1/a$ i.e. $1/a = 1/\lambda$ such that $\lambda = a$

$\therefore$ The probability density function for the exponential distribution is $f(t) = a e^{-at}$

$$\Pr(0 \le T \le x) = \int_0^x a e^{-at}\, dt$$

$$= a \left[-\frac{e^{-at}}{a} + k \right]_0^x$$

$$= 1 - e^{-ax}$$

i.e. $\Pr(0 \le T \le x) = 1 - e^{-ax}$

$$\Pr(x_1 \le T \le x_2) = \int_{x_1}^{x_2} a e^{-at}\, dt$$

$$= \left[\frac{-a e^{-at}}{a} + k \right]_{x_1}^{x_2}$$

$$= e^{-ax_1} - e^{-ax_2}$$

$$Pr(T \geq x) = \int_x^{\infty} a e^{-at}\, dt$$

$$= \left[\frac{-a e^{-at}}{a} + k \right]_x^{\infty}$$

$$= 0 + e^{-ax}$$

$$= e^{-ax}$$

---

11) The mean $\mu = 2000$ hours and so $\lambda = \frac{1}{2000}$ such that $f(x) = \lambda e^{-\lambda x}$

$$= \frac{e^{-x/2000}}{2000}$$

a) $Pr(X \leq 2400)$

$$= \int_0^{2400} \frac{e^{-x/2000}}{2000}\, dx$$

$$= \frac{1}{2000} \int_0^{2400} e^{-x/2000}\, dx$$

$$= \frac{1}{2000} \left[-2000\, e^{-x/2000} + k \right]_0^{2400}$$

$$= -e^{-2400/2000} + e^{0}$$

$$= e^{0} - e^{-1.2}$$

$$= 1 - 0.3011942$$

$$= 0.6988057$$

i.e. $Pr(X \leq 2400) = 0.6988057$

b) $Pr(X \geq 1600)$

$$= \int_{1600}^{\infty} \frac{1}{2000} e^{-x/2000}\, dx$$

$$= \frac{1}{2000} \int_{1600}^{\infty} e^{-x/2000}\, dx$$

$$= \frac{1}{2000}\left[-2000\, e^{-x/2000} + k\right]_{1600}^{x}$$

$$= -e^{-x} + e^{-\frac{1600}{2000}}$$

$$= e^{-0.8} \quad \text{as} \quad e^{-\infty} = 0$$

$$= 0.4493284$$

$$\therefore \Pr(X \geq 1600) = 0.4493284$$

c) $\Pr(1800 < X < 2200)$

$$= \int_{1800}^{2200} \frac{e^{-x/2000}}{2000}\, dx$$

$$= \frac{1}{2000} \int_{1800}^{2200} e^{-x/2000}\, dx$$

$$= \frac{1}{2000}\left[-2000\, e^{-x/2000} + k\right]_{1800}^{2200}$$

$$= -e^{-\frac{2200}{2000}} + e^{-\frac{1800}{2000}}$$

$$= e^{-0.9} - e^{-1.1}$$

$$= 0.4065696 - 0.332871$$

$$= 0.0736985$$

$$\therefore \Pr(1800 < X < 2200) = 0.0736985$$

Tutorial Sheet No 7

2

Mean $\mu = (1+4+7+8)/4$

$= 20/4$

$= 5$

Variance $\sigma^2 = \frac{1}{4}\{(1-5)^2+(4-5)^2+(7-5)^2+(8-5)^2\}$

$= \frac{1}{4}\{(-4)^2+(1)^2+(2)^2+(3)^2\}$

$= \frac{1}{4}\{16+1+4+9\}$

$= \frac{30}{4}$ i.e Variance $\sigma^2 = 7.5$

Possible samples of size 2 : (1, 4), (1, 7), (1, 8), (4, 7), (4, 8), (7, 8), (8, 7), (7, 4), (8, 1), (7, 1) (4, 1)} (8, 4)

Mean of Samples	2.5	4	4.5	5.5	6	7.5
Frequency	2	2	2	2	2	2

Mean $\mu_{\bar{x}} = \dfrac{2(2.5+4+4.5+5.5+6+7.5)}{12}$

$= \dfrac{2\times 30}{12}$ i.e $\mu_{\bar{x}} = 5$

Variance $\sigma_{\bar{x}}^2 = 2\{(2.5-5)^2+(4-5)^2+(4.5-5)^2+(5.5-5)^2+(6-5)^2 + (7.5-5)^2\}/12$

$= \dfrac{2\{(-2.5)^2+(-1)^2+(-0.5)^2+(0.5)^2+1^2+(2.5)^2\}}{12}$

$= \dfrac{2\{6.25+1+0.25+0.25+1+6.25\}}{12}$

$= \dfrac{2\times 15}{12}$ $\sigma_{\bar{x}}^2 = 2.5$

Now $\sigma_{\bar{x}}^2 = \dfrac{\sigma^2}{n}\left(\dfrac{N-n}{N-1}\right)$

$= \dfrac{7.5}{2}\left(\dfrac{4-2}{4-1}\right)$

$= \dfrac{7.5}{2} \cdot \dfrac{2}{3}$

$= 2.5$ i.e. $\sigma_{\bar{x}}^2 = 2.5$

$$Var(\bar{X}) = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)$$

$$= \frac{\sigma^2}{n}\left(\frac{\{N-1\}-\{n-1\}}{N-1}\right)$$

$$= \frac{\sigma^2}{n}\left(1 - \frac{\{n-1\}}{(N-1)}\right)$$

As $N \to \infty$, $Var(\bar{X}) \to \frac{\sigma^2}{n}$ as $\left(\frac{n-1}{N-1}\right) \to 0$ as $N \to \infty$

---

3. a) $Pr(\bar{X} < 48.5)$

$X \sim N(50, 12)$

$\bar{X} \sim N(50, 12/12)$

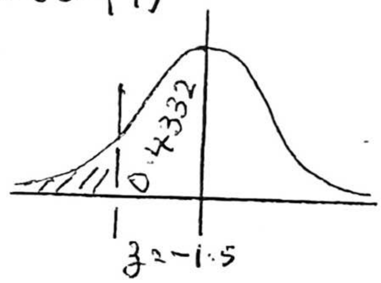
$= N(50, 1)$

$= Pr\left(\frac{\bar{X}-50}{1} < \frac{48.5-50}{1}\right)$

$= Pr(Z < -1.5)$

$= 0.5 - 0.4332$

$= 0.0668$



$\therefore Pr(\bar{X} < 48.5) = 0.0668$

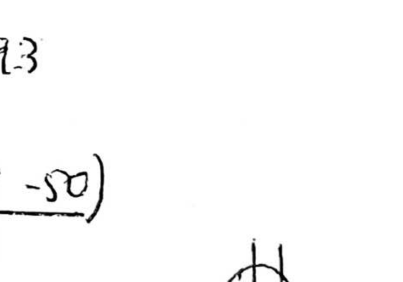
b) $Pr(\bar{X} < 52.3)$

$= Pr\left(\frac{\bar{X}-50}{1} < \frac{52.3-50}{1}\right)$

$= Pr(Z < 2.3)$

$= 0.5 + 0.4893$

$= 0.9893$



$\therefore Pr(\bar{X} < 52.3) = 0.9893$

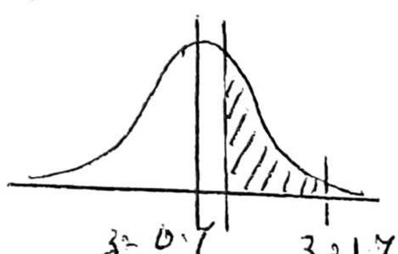
c) $Pr(50.7 < \bar{X} < 51.7)$

$= Pr\left(\frac{50.7-50}{1} < \frac{\bar{X}-50}{1} < \frac{51.7-50}{1}\right)$

$= Pr(0.7 < Z < 1.7)$

$= 0.4554 - 0.2580$

$= 0.1974$



$\therefore \Pr(50.4 < \bar{X} < 51.7) = 0.1974$

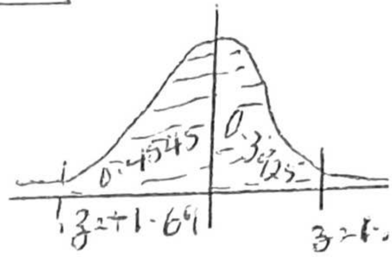
d) $\Pr(48.31 < \bar{X} < 51.24)$

$= \Pr\left(\frac{48.31-50}{1} < \frac{\bar{X}-50}{1} < \frac{51.24-50}{1}\right)$

$= \Pr(-1.69 < Z < 1.24)$

$= 0.4545 + 0.3925$

$= 0.8470$



$\therefore \Pr(48.31 < \bar{X} < 51.24) = 0.8470$

---

4. $X \sim N(52.5, 6.5^2)$ and $\bar{X} \sim N(52.5, 6.5^2/20)$

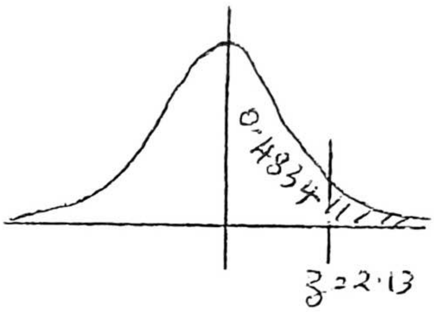
i) $\Pr(\bar{X} > 55.6)$

$= \Pr\left(\frac{\bar{X}-52.5}{6.5/\sqrt{20}} > \frac{55.6-52.5}{6.5/\sqrt{20}}\right)$

$= \Pr\left(Z > \frac{3.1}{6.5/\sqrt{20}}\right)$

$= \Pr(Z > 2.13286484)$

$= \Pr(Z > 2.13)$



$= 0.5 - 0.4834$

$= 0.0166$ i.e $\Pr(\bar{X} > 55.6) = 0.0166$

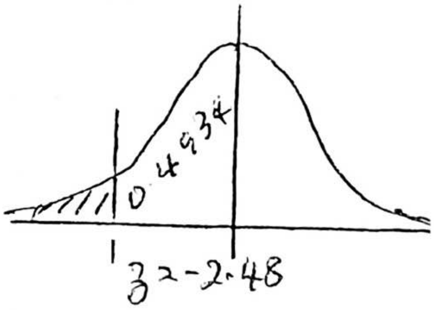
ii) $\Pr(\bar{X} < 48.9)$

$= \Pr\left(\frac{\bar{X}-52.5}{6.5/\sqrt{20}} < \frac{48.9-52.5}{6.5/\sqrt{20}}\right)$

$= \Pr\left(Z < \frac{-3.6}{6.5/\sqrt{20}}\right)$

$= \Pr(Z < -2.476875298)$

$= \Pr(Z < -2.48)$



$= 0.5 - 0.4934$

$= 0.0066$ i.e $\Pr(\bar{X} < 48.9) = 0.0066$

iii) $\Pr(49.7 < \bar{X} < 54.9)$

$= \Pr\left(\frac{49.7-52.5}{6.5/\sqrt{20}} < \frac{\bar{X}-52.5}{6.5/\sqrt{20}} < \frac{54.9-52.5}{6.5/\sqrt{20}}\right)$

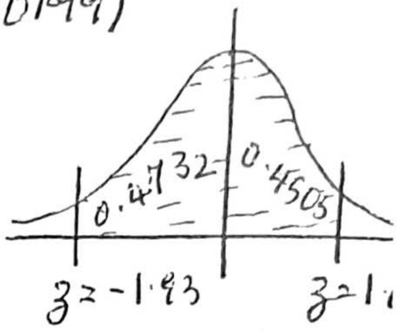
$$= Pr\left(\frac{-2.8}{6.5/\sqrt{20}} < z < \frac{2.4}{6.5/\sqrt{20}}\right)$$

$$= Pr(-1.926458565 < z < 1.651250199)$$

$$= Pr(-1.93 < z < 1.65)$$

$$= 0.4732 + 0.4505$$

$$= 0.9237$$



i.e $Pr(49.7 < \bar{X} < 54.9) = 0.9237$

iv) $Pr(54.3 < \bar{X} < 56.9)$

$$= Pr\left(\frac{54.3 - 52.5}{6.5/\sqrt{20}} < \frac{X - 52.5}{6.5/\sqrt{20}} < \frac{56.9 - 52.5}{6.5/\sqrt{20}}\right)$$

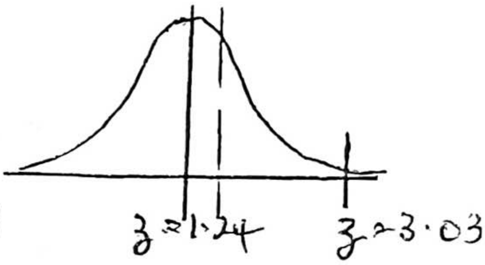
$$= Pr\left(\frac{1.8}{6.5/\sqrt{20}} < z < \frac{4.4}{6.5/\sqrt{20}}\right)$$

$$= Pr(1.238437649 < z < 3.027292031)$$

$$= Pr(1.24 < z < 3.03)$$

$$= 0.4988 - 0.3925$$

$$= 0.1063$$



i.e $Pr(54.3 < \bar{X} < 56.9) = 0.1063$

---

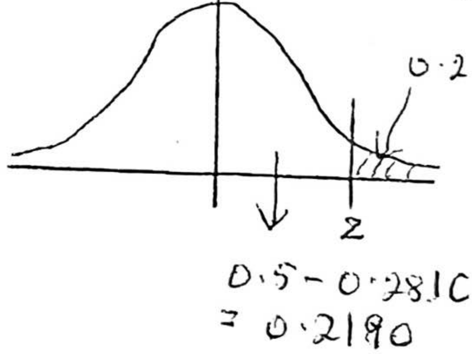
5 $X \sim N(74, 36)$ and $\bar{X} \sim N(74, 36/n)$

a) $Pr(\bar{X} > 75) = 0.2810$

$$Pr\left(\frac{\bar{X} - 74}{6/\sqrt{n}} > \frac{75 - 74}{6/\sqrt{n}}\right) = 0.2810$$

$$Pr\left(z > \frac{1}{6/\sqrt{n}}\right) = 0.2810$$

$$Pr\left(z > \frac{\sqrt{n}}{6}\right) = 0.2810$$



From tables z : 0.58

$$\therefore 0.58 > \frac{\sqrt{n}}{6}$$

$$\sqrt{n} < 0.58 \times 6$$

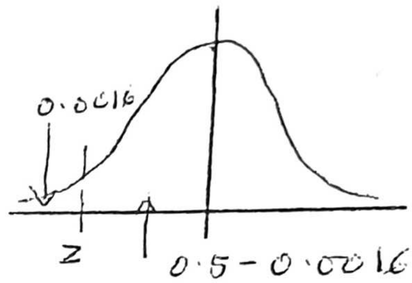
$n < (0.58 \times 6)^2$

$n < 12.1104$

$\therefore n = 12$

b) $Pr(\bar{X} < 70.4) = 0.0016$

$Pr\left(\frac{\bar{X} - 74}{6/\sqrt{n}} < \frac{70.4 - 74}{6/\sqrt{n}}\right) = 0.0016$



$Pr\left(z < \frac{-3.6}{6/\sqrt{n}}\right) = 0.0016$

$Pr\left(z < -\frac{3.6\sqrt{n}}{6}\right) = 0.0016$

$Pr(z < -0.6\sqrt{n}) = 0.0016$

From tables $z = -2.95$

$\therefore -2.95 < -0.6\sqrt{n}$

$2.95 > 0.6\sqrt{n}$

$\sqrt{n} < \frac{2.95}{0.6}$

$n < \left(\frac{2.95}{0.6}\right)^2$

$= 24.173611$

$\therefore n = 24$

---

6 X is the size of pebbles in a river bed in United Kingdom.

$X \sim N(12.7, 3.1^2)$ and $\bar{X} \sim N(12.7, 3.1^2/n)$

a) $Pr(\bar{X} < 13.5) = 0.8686$

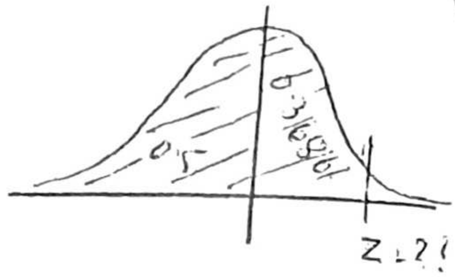
$Pr\left(\frac{\bar{X} - 12.7}{3.1/\sqrt{n}} < \frac{13.5 - 12.7}{3.1/\sqrt{n}}\right) = 0.8686$

$Pr\left(z < \frac{0.8}{3.1/\sqrt{n}}\right) = 0.8686$

$Pr\left(z < \frac{0.8\sqrt{n}}{3.1}\right) = 0.8686$

From tables $z = 1.12$

$\therefore 1.12 < \dfrac{0.8\sqrt{n}}{3.1}$



$\dfrac{1.12 \times 3.1}{0.8} < \sqrt{n}$

$n > \left(\dfrac{1.12 \times 3.1}{0.8}\right)^2$

$= (4.34)^2$

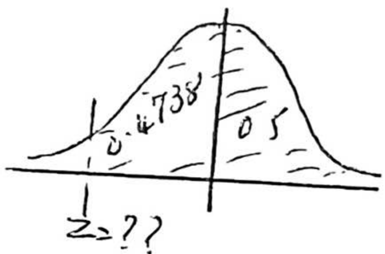
$= 18.8356$

Smallest value of $n = 19$.

b) $Pr(\bar{X} > 10.9) = 0.9738$

$Pr\left(\dfrac{\bar{X} - 12.7}{3.1/\sqrt{n}} > \dfrac{10.9 - 12.7}{3.1/\sqrt{n}}\right) = 0.9738$

$Pr\left(z > \dfrac{-1.8}{3.1/\sqrt{n}}\right) = 0.9738$



$Pr\left(z > \dfrac{-1.8\sqrt{n}}{3.1}\right) = 0.9738$

From tables $z = -1.94$

$-1.94 > \dfrac{-1.8\sqrt{n}}{3.1}$

$1.94 < \dfrac{1.8\sqrt{n}}{3.1}$

$\sqrt{n} > \dfrac{1.94 \times 3.1}{1.8}$

$n > \left(\dfrac{1.94 \times 3.1}{1.8}\right)^2$

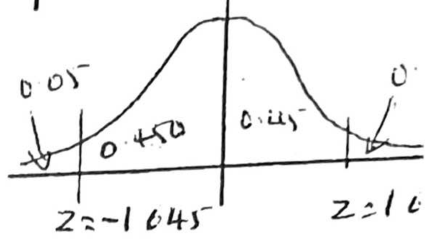
$= 11.16302348$

Smallest value of $n = 12$

Tutorial Sheet No 8

1 a) 90% confidence interval for pop. mean μ

$= \bar{x} \pm Z_{0.05} \frac{\sigma}{\sqrt{n}}$



$= 76 \pm 1.645 \times \frac{12}{\sqrt{100}}$

$= 76 \pm 1.645 \times 1.2$

$= 76 \pm 1.974$

$= 76 - 1.974,\ 76 + 1.974$

$= 74.026,\ 77.974$

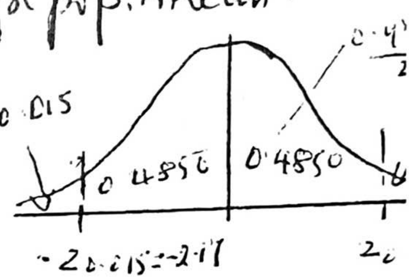
$-1.645 < z < 1.645$

$-1.645 < \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} < 1.645$

when $\sigma_{\bar{x}} =$

b) 97% confidence interval for pop. mean

$= \bar{x} \pm Z_{0.015} \frac{\sigma}{\sqrt{n}}$



$= 76 \pm 2.17 \times \frac{12}{\sqrt{100}}$

$= 76 \pm 2.17 \times 1.2$

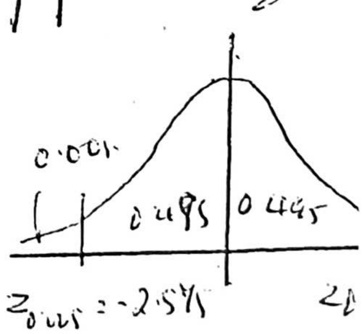
$= 76 \pm 2.604$

$= 76 - 2.604,\ 76 + 2.604$

$= 73.396,\ 78.604$

c) 99% confidence interval for pop. mean μ

$= \bar{x} \pm Z_{0.005} \frac{\sigma}{\sqrt{n}}$



$= 76 \pm 2.575 \times \frac{12}{\sqrt{100}}$

$= 76 \pm 2.575 \times 1.2$

$= 76 \pm 3.09$

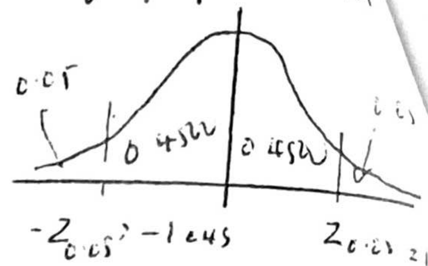
$= 76 - 3.09,\ 76 + 3.09$

$= 72.91,\ 79.09$

2

a) 90% confidence interval for pop. mean

$= \bar{x} \pm Z_{0.05} \frac{\sigma}{\sqrt{n}}$



$= 748 \pm 1.645 \times \frac{3.6}{\sqrt{150}}$

$= 748 \pm 1.645 \times 0.293938769$

$= 748 \pm 0.483529279$

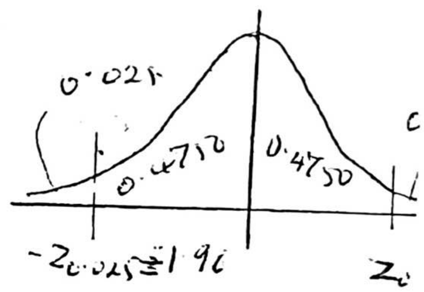
$= 748 - 0.483529279,\ 748 + 0.483529279$

$= 747.5164707,\ 748.4835293$

$= 747.5165,\ 748.4835$

b) 95% confidence interval for pop. mean

$= \bar{x} \pm Z_{0.025} \frac{\sigma}{\sqrt{n}}$



$= 748 \pm 1.96 \times \frac{3.6}{\sqrt{150}}$

$= 748 \pm 1.96 \times 0.29393876$

$= 748 \pm 0.576119969$

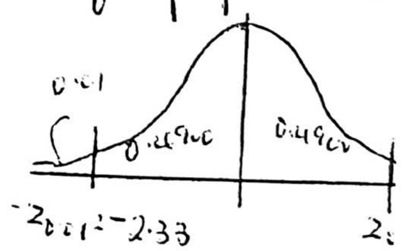
$= 748 - 0.576119969,\ 748 + 0.576119969$

$= 747.42388,\ 748.576119969$

$= 747.4239,\ 748.57612$

c) 98% confidence interval for pop. mean

$= \bar{x} + Z_{0.01} \frac{\sigma}{\sqrt{n}}$



$= 748 \pm 2.33 \times \frac{3.6}{\sqrt{150}}$

$= 748 \pm 2.33 \times 0.29393876$

$= 748 \pm 0.68487731$

$= 748 - 0.68487731,\ 748.68487731$

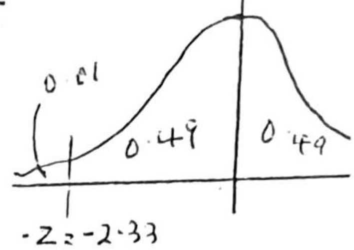
$= 747.3151227,\ 748.6847731$

$= 747.3151,\ 748.6848$

3 a) 98% confidence interval for the difference in average temperatures at the two locations

$= (\bar{x}_A - \bar{x}_B) \pm Z_{0.01}\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}$

$= (59.5 - 57.3) \pm 2.33\sqrt{\frac{6.1^2}{52} + \frac{5.9^2}{55}}$



$= 2.2 \pm 2.33\sqrt{\frac{37.21}{52} + \frac{34.81}{55}}$

$= 2.2 \pm 2.33\sqrt{0.715576923 + 0.63290909}$

$= 2.2 \pm 2.33\sqrt{1.348486013}$

$= 2.2 \pm 2.33 \times 1.161243309$

$= 2.2 \pm 2.70569690$

$= 2.2 - 2.70569690,\ 2.2 + 2.70569690$

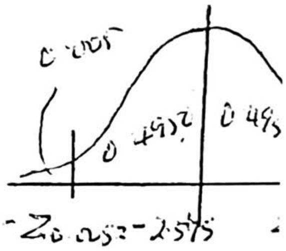
$= -0.50569690,\ 4.90569690$

$= -0.5057,\ 4.9057$

b) 99% confidence interval for the difference the average temperatures at the two locations

$= (\bar{x}_A - \bar{x}_B) \pm Z_{0.005}\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}$

$= (59.5 - 57.3) \pm 2.575\sqrt{\frac{6.1^2}{52} + \frac{5.9^2}{55}}$



$= 2.2 \pm 2.575\sqrt{1.348486013}$

$= 2.2 \pm 2.575 \times 1.161243309$

$= 2.2 \pm 2.990201521$

$= 2.2 - 2.990201521,\ 2.2 + 2.990201521$

$= -0.790201521,\ 5.190201521$

$= -0.7902,\ 5.1902$

---

4 95% confidence interval for the difference between the means of banking and insurance

executives is $(\bar{x}_I - \bar{x}_B) \pm Z_{0.025}\sqrt{\frac{\sigma_I^2}{n_I} + \frac{\sigma_B^2}{n_B}}$

$= (9.4 - 6.5) \pm 1.96\sqrt{\frac{3.3^2}{40} + \frac{2.9^2}{50}}$

$= 2.9 \pm 1.96\sqrt{\frac{10.89}{40} + \frac{8.41}{50}}$

$= 2.9 \pm 1.96\sqrt{0.27225 + 0.1682}$

$= 2.9 \pm 1.96\sqrt{0.44045}$

$= 2.9 \pm 1.96 \times 0.663664071$

$= 2.9 \pm 1.30078158$

$= 2.9 - 1.30078158,\ 2.9 + 1.30078158$

$= 1.59921842,\ 4.20078158$

$= 1.5992,\ 4.2008$

---

5 Mean $\bar{x}$

$= \sum_{i=1}^{10} x_i/10$

$= \{1.504 + 1.496 + 1.492 + 1.501 + 1.503 + 1.505 + 1.4$

$+ 1.502 + 1.493 + 1.501\}/10$

$= 14.990/10$

$= 1.499$ i.e $\bar{x} = 1.499$

Variance s^2

$= \sum_{i=1}^{10}(x_i - 1.499)^2/9$

$= \{(1.504 - 1.499)^2 + (1.496 - 1.499)^2 + (1.492 - 1.49$

$(1.501 - 1.499)^2 + (1.503 - 1.499)^2 + (1.505 - 1.499)^2 + (1.49$

$1.499)^2 + (1.502 - 1.499)^2 + (1.493 - 1.499)^2 +$

$(1.501 - 1.499)^2\}/9$

$= \{(0.005)^2 + (-0.003)^2 + (-0.007)^2 + (0.002$

$(0.004)^2 + (0.006)^2 + (-0.004)^2 + (0.001)^2 + (-0.006$

$+ (0.002)^2\}/9$

$= \{0.000025 + 0.000009 + 0.000049 + 0.000004$

$0.000016 + 0.000036 + 0.000016 + 0.000001 + 0.000036$
$0.000004\}/9$

$= 0.000196/9.$

95% confidence interval for population mean

$= \bar{x} \pm t_{0.025,9} \frac{s}{\sqrt{n}}$

$= 1.499 \pm 2.26 \frac{\sqrt{0.000196/9}}{\sqrt{10}}$

$= 1.499 \pm 2.26 \sqrt{\frac{0.000196}{90}}$

$= 1.499 \pm 2.26 \times 0.001475729$

$= 1.499 \pm 0.003335148$

$= 1.499 - 0.003335148,\ 1.499 + 0.003335148$

$= 1.495664852,\ 1.502335149$

$= 1.4957,\ 1.5023$

---

6 Mean $\bar{x}$

$= \sum_{i=1}^{6} x_i/6$

$= \{21 + 26 + 24 + 22 + 23 + 22\}/6$

$= 138/6$

$= 23$ i.e $\bar{x} = 23$

Variance s^2

$= \sum_{i=1}^{6} (x_i - \bar{x})^2/5$

$= \{(21-23)^2 + (26-23)^2 + (24-23)^2 + (22-23)$
$(23-23)^2 + (22-23)^2\}/5$

$= \{(-2)^2 + (3)^2 + (1)^2 + (-1)^2 + (0)^2 + (-1)^2\}/5$

$= \{4 + 9 + 1 + 1 + 0 + 1\}/5$

$= 16/5$

$= 3.2$ i.e $s^2 = 3.2$

95% confidence interval for pop. mea

$$= \bar{x} + t_{0.025, 5} \frac{s}{\sqrt{n}}$$

$$= 23 \pm 2.57 \frac{\sqrt{3.2}}{\sqrt{6}}$$

$$= 23 \pm 2.57 \sqrt{3.2/6}$$

$$= 23 \pm 2.57 \times 0.730296743$$

$$= 2.3 \pm 1.87686263$$

$$= 23 - 1.87686263,\ 23 + 1.87686263$$

$$= 21.12313737,\ 24.87686263$$

$$= 21.1231,\ 24.8769$$

---

7 $Z_{0.025} = 1.96$, $\sigma = 120$ and error $E = 20$

$$E \le \frac{Z_{0.025}\,\sigma}{\sqrt{n}}$$

$$\sqrt{n} \le \frac{Z_{0.025}\,\sigma}{E}$$

$$= \left(\frac{1.96 \times 120}{20}\right)$$

$$\therefore n \le (11.76)^2$$

$$= 138.2976$$

Thus the sample should consist of at most 138 students.

---

8 $Z_{0.05} = 1.645$, $E = 25$ and $\sigma = 150$

$$E \le \frac{Z_{0.05}\,\sigma}{\sqrt{n}}$$

$$\sqrt{n} \le \frac{Z_{0.05}\,\sigma}{E}$$

$$= \frac{1.645 \times 150}{25}$$

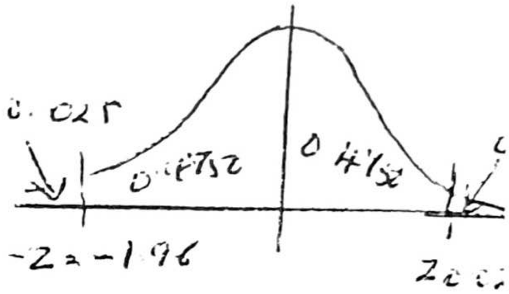
$$n \le \left(\frac{1.645 \times 150}{25}\right)^2$$
$$= (9.87)^2$$
$$= 97.4169$$

The sample should be made up of 97 person most

---

9 a) $p_s = \frac{144}{200}$

$= 0.72$



$q_s = 1 - p_s$

$= 1 - 0.72$

$= 0.28$

95% confidence interval for the population proportion of those who opposed an increase the humanities requirements

$$= p_s \pm Z_{0.025}\sqrt{\frac{p_s q_s}{n}}$$
$$= 0.72 \pm 1.96\sqrt{\frac{0.72 \times 0.28}{200}}$$
$$= 0.72 \pm 1.96\sqrt{1.008 \times 10^{-3}}$$
$$= 0.72 \pm 1.96 \times 0.031749015$$
$$= 0.72 \pm 0.06222807$$
$$= 0.72 - 0.06222807,\ 0.72 + 0.06222807$$
$$= 0.657771929,\ 0.78222807$$
$$= 0.6578,\ 0.7822$$

b) $p_s = 0.28$ and $q_s = 0.72$

95% confidence interval for the population proportion of those who are in favour of an inc in the humanities requirements

$= p_s \pm Z_{0.025}\sqrt{\dfrac{p_s q_s}{n}}$

$= 0.28 \pm 1.96\sqrt{\dfrac{0.28 \times 0.72}{200}}$

$= 0.28 \pm 0.06222807$

$= 0.28 - 0.06222807,\ 0.28 + 0.06222807$

$= 0.21777193,\ 0.34222807$

$= 0.2178,\ 0.3422$

---

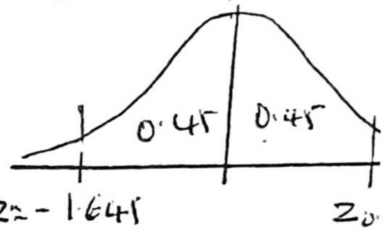
10 $p_s = \dfrac{30}{100}$ $\qquad q_s = 1 - p_s$

$= 0.3 \qquad\qquad = 1 - 0.3$

$\qquad\qquad\qquad = 0.7$

a) 90% confidence interval for true proport

$= p_s \pm Z_{0.05}\sqrt{\dfrac{p_s q_s}{n}}$



$= 0.3 \pm 1.645\sqrt{\dfrac{0.3 \times 0.7}{100}}$

$= 0.3 \pm 1.645\sqrt{0.0021}$

$= 0.3 \pm 1.645 \times 0.045825756$

$= 0.3 \pm 0.07538337$

$= 0.3 - 0.07538337,\ 0.3 + 0.07538337$

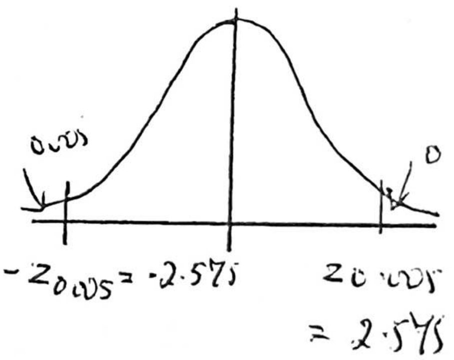
$= 0.22461663,\ 0.37538337$

$= 0.2246,\ 0.3754$

b) 99% confidence interval for true proporti

$= p_s \pm Z_{0.005}\sqrt{\dfrac{p_s q_s}{n}}$

$= 0.3 \pm 2.575\sqrt{\dfrac{0.3 \times 0.7}{100}}$



$= 0.3 \pm 2.575 \times 0.045825756$

$= 0.3 \pm 0.118001324$

$= 0.3 - 0.118001324, 0.3 + 0.118001324$

$= 0.181998679, 0.418001324$

$= 0.1820, 0.4180$

11

$p = 0.63$

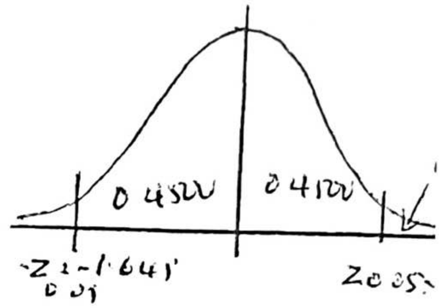
$q = 1 - p$

$= 1 - 0.63$

$= 0.37$

a) $E = 0.05$

$E \le Z_{0.05}\sqrt{\frac{pq}{n}}$



$\sqrt{n} \le \frac{Z_{0.05}\sqrt{pq}}{E}$

$= \frac{1.645\sqrt{0.63 \times 0.37}}{0.05}$

$\therefore n \le \left(\frac{1.645\sqrt{0.63 \times 0.37}}{0.05}\right)^2$

$= \left(\frac{1.645\sqrt{0.2331}}{0.05}\right)^2$

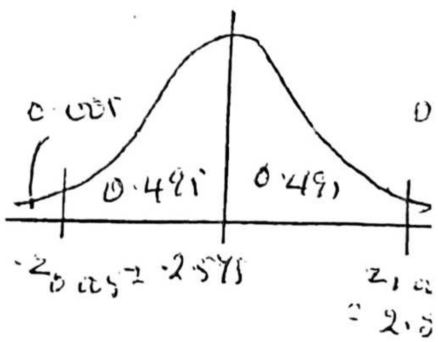
$= \left(\frac{1.645 \times 0.482804308}{0.05}\right)^2$

$= \left(\frac{0.794213308}{0.05}\right)^2$

$= (15.88426174)^2$

$= 252.309771$ i.e. $n = 252$

b) $E \le Z_{0.005}\sqrt{\frac{pq}{n}}$



$\sqrt{n} \le \frac{Z_{0.005}\sqrt{pq}}{E}$

$= \frac{2.575\sqrt{0.63 \times 0.37}}{0.05}$

$$n \le \left(\frac{2.575\sqrt{0.63 \times 0.37}}{0.05}\right)^2$$

$$= \left(\frac{2.575 \times 0.482804308}{0.05}\right)^2$$

$$= \left(\frac{1.243221094}{0.05}\right)^2$$

$$= (24.86442187)^2$$

$$= 618.239475 \quad \therefore n = 618$$

12

$p_1 = \frac{57}{150}$ $\quad$ $q_1 = 1 - p_1$

$= 0.38$ $\quad$ $= 1 - 0.38$

$\quad$ $= 0.62$

$p_2 = \frac{33}{100}$ $\quad$ $q_2 = 1 - p_2$

$= 0.33$ $\quad$ $= 1 - 0.33$

$\quad$ $= 0.67$

95% confidence interval for the difference between the true proportions

$$= (p_1 - p_2) \pm Z_{0.025}\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$$

$$= (0.38 - 0.33) \pm 1.96\sqrt{\frac{0.38 \times 0.62}{150} + \frac{0.33 \times 0.67}{100}}$$

$$= 0.05 \pm 1.96\sqrt{\frac{0.2356}{150} + \frac{0.2211}{100}}$$

$$= 0.05 \pm 1.96\sqrt{1.570666667 \times 10^{-3} + 2.211 \times 10^{-3}}$$

$$= 0.05 \pm 1.96\sqrt{3.781666667 \times 10^{-3}}$$

$$= 0.05 \pm 1.96 \times 0.061495257$$

$$= 0.05 \pm 0.120530704$$

$$= 0.05 - 0.120530704,\ 0.05 + 0.120530704$$

$$= -0.070530704,\ 0.170530704$$

$= -0.0705, 0.1705$

13 $p_1 = \frac{102}{250}$ $q_1 = 1 - p_1$

$= 0.408$ $= 1 - 0.408$

$= 0.592$

$p_2 = \frac{73}{250}$ $q_2 = 1 - p_2$

$= 1 - 0.292$

$= 0.292$ $= 0.708$

99% confidence interval for the difference between the true proportions

$= (p_1 - p_2) \pm Z_{0.005}\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$

$= (0.408 - 0.292) \pm 2.575\sqrt{\frac{0.408 \times 0.592}{250} + \frac{0.292 \times 0.708}{250}}$

$= 0.116 \pm 2.575\sqrt{\frac{0.241536}{250} + \frac{0.206736}{250}}$

$= 0.116 \pm 2.575\sqrt{9.66144 \times 10^{-4} + 8.26944 \times 10^{-4}}$

$= 0.116 \pm 2.575\sqrt{17.93088 \times 10^{-4}}$

$= 0.116 \pm 2.575 \times 0.042344869$

$= 0.116 \pm 0.109038039$

$= 0.116 - 0.109038039, 0.116 + 0.109038039$

$= 0.006961960216, 0.225038039$

$= 0.0070, 0.2250$

Tutorial Sheet No 9

1 $H_0: T_A = T_B$ and then $Pr(A \text{ wins}) = 1/2$ and $Pr(B \text{ wins}) = 1/2$

$H_1: T_A \neq T_B$ and then $Pr(A \text{ wins}) = 11/15$ and $Pr(B \text{ wins}) = 4/15$

H_0 is accepted if B obtains a faster time than A in any race of the three races. accept H_1 | H_0 is true.

$Pr(\text{Type I Error}) = Pr(\text{Reject } H_0 \mid H_0 \text{ is true})$

$= Pr(A \text{ wins 3 races} \mid H_0 \text{ is true})$

$= \left(\frac{1}{2}\right)^3$

$= \frac{1}{8} = 0.125$

or H_1 is true

$Pr(\text{Type II Error}) = Pr(\text{Accept } H_0 \mid H_0 \text{ is false})$

$= Pr(B \text{ wins at least one race} \mid H_0 \text{ is false})$

$= 1 - \left(\frac{11}{15}\right)^3$

$= 1 - 0.3943703$

$= 0.6056296$

---

2 $H_0: \mu = 65$ and $H_1: \mu > 65$

H_0 is rejected when $\bar{x} > 66$

a) $Pr(\text{Type I Error})$

$= Pr(\text{Reject } H_0 \mid H_0 \text{ is true})$

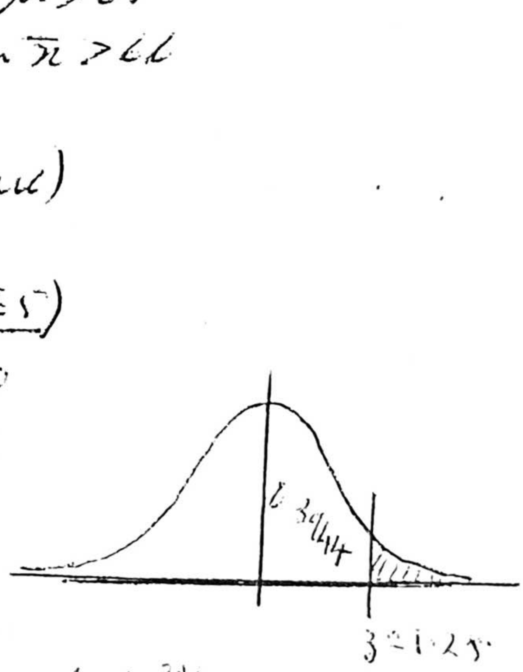
$= Pr(\bar{X} > 66)$

$= Pr\left(\frac{\bar{X} - 65}{8/\sqrt{100}} > \frac{66 - 65}{8/10}\right)$

$= Pr\left(Z > \frac{1}{4/5}\right)$

$= Pr\left(Z > \frac{5}{4}\right)$

$= Pr(Z > 1.25)$ $z = 1.25 \to 0.3944$



$= 0.5 - 0.3944$

$= 0.1056$

$\therefore Pr(\text{Type I Error}) = 0.1056$

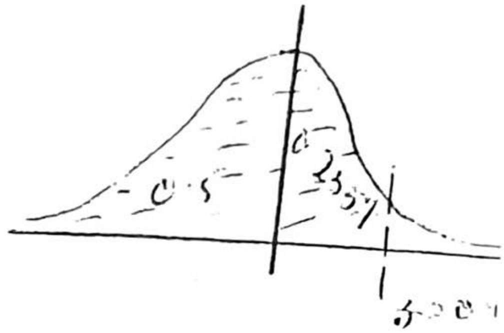
b) $Pr(\text{Type II Error})$

$= Pr(\text{Accept } H_0 \mid H_0 \text{ is false})$ $H_0: \mu = 65.50$ versus $H_1: \mu < 65.50$

$= Pr(\bar{X} < 66 \mid H_0 \text{ is false})$

$= Pr\left(\frac{\bar{X} - 65.5}{8/10} < \frac{66 - 65.5}{8/10}\right)$

$= Pr\left(Z < \frac{0.5}{0.8}\right)$



$= Pr(Z < 0.625)$

$= Pr(Z < 0.63)$

$= 0.5 + 0.2357$

$= 0.7357$

$\therefore Pr(\text{Type II Error}) = 0.7357$

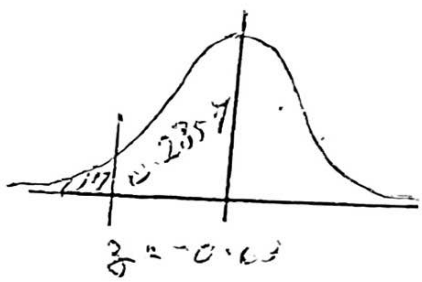
c) $Pr(\text{Type II Error})$

$= Pr(\text{Accept } H_0 \mid H_0 \text{ is false})$

$= Pr(\bar{X} < 66 \mid H_0 \text{ is false})$

$= Pr\left(\frac{\bar{X} - 66.50}{8/10} < \frac{66 - 66.50}{8/10}\right)$

$= Pr\left(Z < -\frac{0.5}{0.8}\right)$



$= Pr(Z < -0.625)$

$= Pr(Z < -0.63)$

$= 0.5 - 0.2357$

$= 0.2643$

$\therefore Pr(\text{Type II Error}) = 0.2643$

3 H_0: The man can throw a six four times of six

H_1: The man can throw a six once out of six for a Hundred four times.

Ho is rejected if the number of six obtained is less than 3.

Binomial distribution is used in this problem. For Ho, Pr(a six) = 2/3 and for H_1, Pr(a six) = ...

Pr(Type I Error) Probability of success = $\frac{2}{3}$

$\frac{4}{6}$ = Pr(Reject Ho | Ho is true) Probability of failure ... $\frac{1}{3}$

= Pr(X = 0) + Pr(X = 1) + Pr(X = 2)

$$= \left(\frac{1}{3}\right)^6 + 6\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right) + \frac{6.5}{1.2}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2$$

${}^6C_0 (1-p)^n \quad {}^6C_1 (1-p)^{n-1} p^1 \quad {}^6C_2 (1-p)^{n-2} p^2 \quad 4!\,2!$

$$= \frac{1}{729} + \frac{12}{729} + \frac{60}{729}$$

$$= \frac{73}{729}$$

= 0.100137174

Pr(Type I Error) = 0.100137174

Pr(Type II Error)

= Pr(Accept Ho | Ho is false)

= Pr(X = 3) + Pr(X = 4) + Pr(X = 5) + Pr(X = 6)

$$= \frac{6.5.4}{1.2.3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^3 + \frac{6.5}{1.2}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^2 + 6\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right) + \left(\frac{1}{6}\right)^6$$

${}^6C_6 = 1$

$$= \frac{20 \times 125}{46656} + \frac{15 \times 25}{46656} + \frac{30}{46656} + \frac{1}{46656}$$

$$= \frac{2500}{46656} + \frac{375}{46656} + \frac{30}{46656} + \frac{1}{46656}$$

$$= \frac{2906}{46656}$$

= 0.062285665

Pr(Type II Error)

= 0.062285665

$= 1 - Pr(\text{Type II Error})$

$= 1 - 0.062285665$

$= 0.937714334$

---

4 $H_0: \mu = \mu_0 (=25)$ and $H_1: \mu > \mu_0$

Test Statistic Z

$$= \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

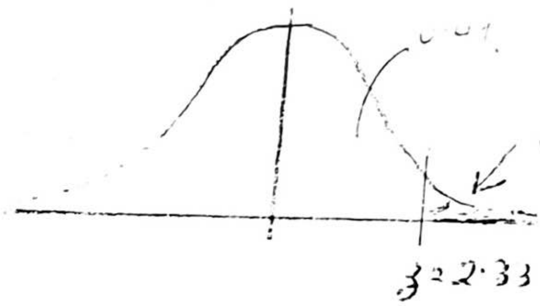
$$= \frac{25.8 - 25}{1.5/\sqrt{32}}$$

$$= \frac{0.8}{1.5/\sqrt{32}}$$

$$= \frac{0.8\sqrt{32}}{1.5}$$

$$= \frac{4.525483}{1.5}$$

$$= 3.016988933$$



The test statistic lies in the critical re.. and so the null hypotheses Ho is rejected.

---

5 $H_0: \mu = \mu_0 (=55)$ $H_1: \mu \neq \mu_0$

Test Statistic Z

$$= \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

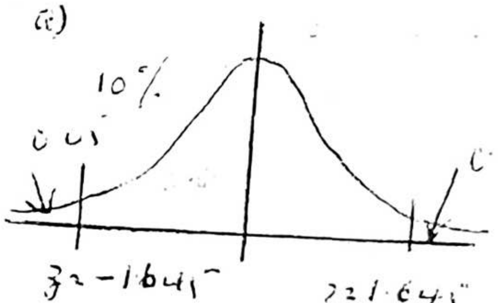
$$= \frac{56.2 - 55}{\sqrt{32}/\sqrt{81}}$$

$$= \frac{1.2}{0.62853936}$$

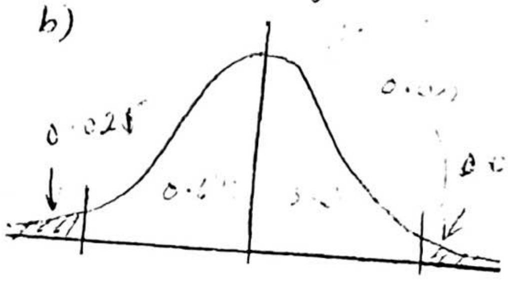
$$= 1.909188309$$

(a) H_0 is rejected as

(a)

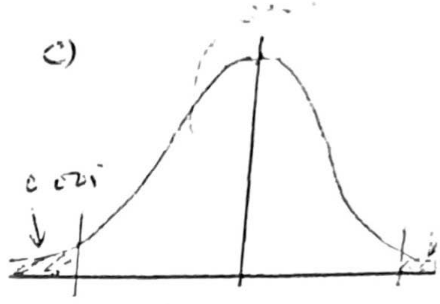


b)



the test statistic lies in
the critical region

b) Ho is accepted as
the test statistic lies in
acceptance region.



c) Ho is accepted as the test statistic lie
the acceptance region

---

6 $H_0: \mu = \mu_0 (= 20)$ and $H_1: \mu \neq \mu_0$

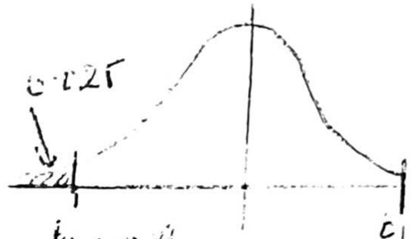
Test Statistic t

$= \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

$= \dfrac{21.9 - 20}{3.4/\sqrt{20}}$

$= \dfrac{1.9}{0.760263112}$

$= 2.499134798$ 2.4



The test statistic lies in the critical region and so the null hypothesis Ho is rej

---

7 $H_0: \mu = \mu_0 (= 10.8)$ $H_1: \mu < \mu_0$

Mean $\bar{x}$

$= (10.7 + 10.65 + 10.75 + 10.8 + 10.6)/5$

$= 53.5/5$

$= 10.7$

Sample variance s^2

$= \sum_{i=1}^{5} (x_i - 10.7)^2/4$

$= \frac{1}{4}\{(10.7-10.7)^2 + (10.65-10.7)^2 + (10.75-10.7)^2 +$

$(10.8-10.7)^2 + (10.6-10.7)^2\}$

$= \frac{1}{4}\{0^2 + (0.05)^2 + (0.05)^2 + (0.1)^2 + (-0.1)^2\}$

$= \frac{1}{4}\{0 + 0.0025 + 0.0025 + 0.01 + 0.01\}$

$= \frac{0.0250}{4}$

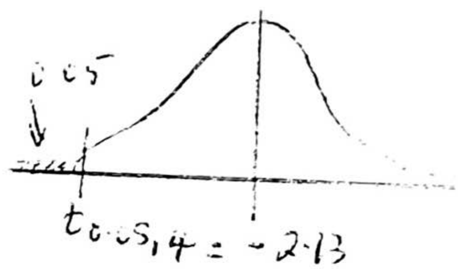
$= 0.00625$ i.e. $s^2 = 0.00625$

Test Statistic

$= \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

$= \frac{10.7 - 10.8}{\sqrt{0.00625}/\sqrt{5}}$

$= \frac{-0.1}{\sqrt{0.00125}}$



$= \frac{-0.1}{0.035355339}$

$= -2.828427125$

The test statistic lies in the critical region and so the null hypothesis H_0 is rejected. This shows that the training has improved his time

8

Sample Mean $\bar{x}$

$= (9.6 + 10.4 + 11.1 + 10.1 + 10.7 + 10.5)/6$

$= \frac{62.4}{6}$

$= 10.4$

Sample Variance s^2

$= \sum_{i=1}^{6} (x_i - 10.4)^2/5$

$= \frac{1}{5}\{(9.6 - 10.4)^2 + (10.4 - 10.4)^2 + (11.1 - 10.4)^2 + ($

$10.4)^2 + (10.7 - 10.4)^2 + (10.5 - 10.4)^2\}$

$= \frac{1}{5}\{(-0.8)^2 + 0^2 + (0.7)^2 + (-0.3)^2 + (0.3)^2 + (0.1$

$= \frac{1}{5}\{0.64 + 0 + 0.49 + 0.09 + 0.09 + 0.01$

$= \frac{1}{5}\{1.32\}$

$= 0.264$ i.e. $s^2 = 0.264$

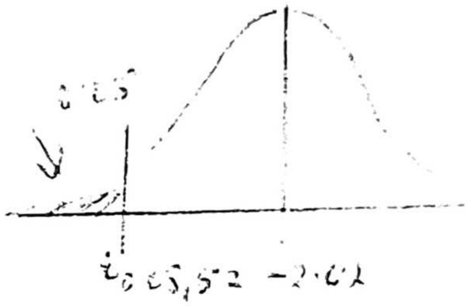
$H_0: \mu = \mu_0 (= 11.0)$ and $H_1: \mu < \mu_0$

Test Statistic

$= \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

$= \frac{10.4 - 11.0}{\sqrt{0.264}/\sqrt{6}}$

$= \frac{-0.6}{\sqrt{0.044}}$



$= \frac{-0.6}{0.209761769}$

$= -2.860387768$

The test statistic lies in the critical region and so the null hypothesis H_0 rejected

---

9. Sample Mean $\bar{x}$

$= \sum_{i=1}^{15} x_i / 15$

$= \frac{57.96}{15}$

$= 3.864$

Sample Variance s^2

$= \frac{1}{n-1}\left\{\sum_{i=1}^{15} x_i^2 - \frac{\left(\sum_{i=1}^{15} x_i\right)^2}{n}\right\}$

$= \frac{1}{14}\{226.1274 - 223.95744\}$

$= \frac{2.16996}{14}$

$= 0.154997142$ i.e. $S^2 = 0.154997142$

i) $H_0: \mu = \mu_0 (= 4.0)$ and $H_1: \mu < \mu_0$

Test Statistic t

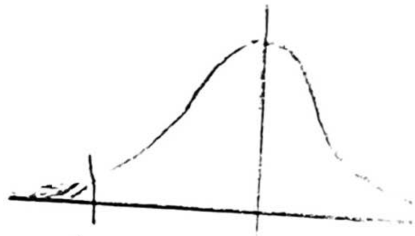
$= \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$

$= \frac{3.864 - 4}{\sqrt{0.154997142}/\sqrt{15}}$

$= \frac{-0.136}{\sqrt{0.010333142}}$

$= \frac{-0.136}{0.101652067}$

$= -1.33789704$



$t_{0.05,14} = -1.76$

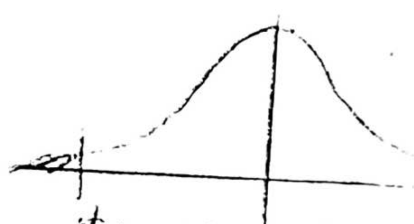
H_0 is accepted as the test statistic lies in the acceptance region

ii) $H_0: \mu = \mu_0$ and $H_1 \neq \mu_0$

Test Statistic t

$= -1.33789704$

H_0 is accepted as the test statistic lies in the acceptance region



$t_{0.025,14} = 2.14$

---

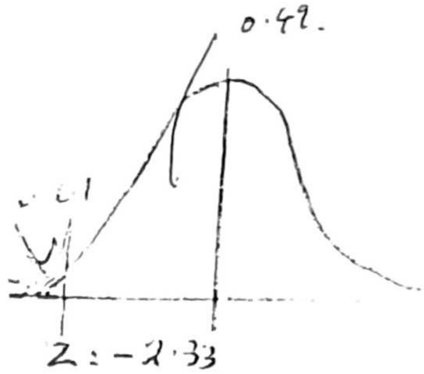
10. $p_s = \frac{140}{400} \Rightarrow p_s = 0.35$

$H_0: p = p_0 (= 0.4)$ and $H_1: p < p_0$

Test Statistic Z

$= \frac{p_s - p_0}{\ldots}$

$$= \frac{0.35 - 0.40}{\sqrt{\frac{0.4(1-0.4)}{400}}}$$



$$= \frac{-0.05}{\sqrt{0.0006}}$$

$$= \frac{-0.05}{0.024494897}$$

$$= -2.04124145 2$$

Ho is accepted as the test statistic lies in the acceptance region.

---

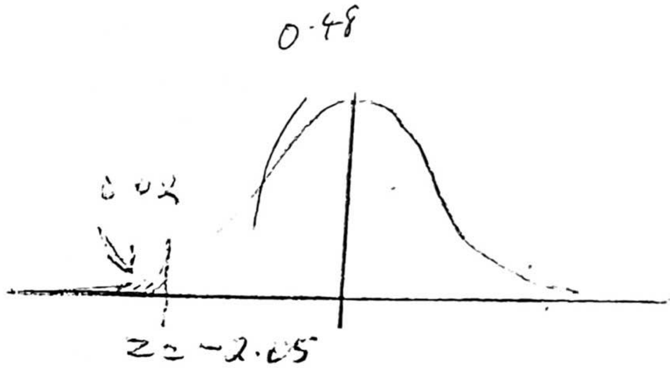
ii) Ho $p = p_0 (= 0.5)$ and H_1 $p < p_0$

$p_s = \frac{38}{100} \Rightarrow p_s = 0.38$

0.5
0.02
0.48

Test Statistic Z

$$= \frac{p_s - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$



$$= \frac{0.38 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{100}}}$$

$$= \frac{-0.12}{\sqrt{0.0025}}$$

$$= \frac{-0.12}{0.05}$$

$$= -2.40$$

The test statistic lies in the critical region and so the null hypothesis Ho is rejected. It's clear that the coin is biased in favour of tails

Tutorial Sheet No10

1

$H_0: \mu_N = \mu_O$ and $H_1: \mu_N > \mu_O$

Test Statistic Z

$$= \frac{(\bar{x}_N - \bar{x}_O) - (\mu_N - \mu_O)}{\sqrt{\frac{\sigma_N^2}{n_N} + \frac{\sigma_O^2}{n_O}}}$$

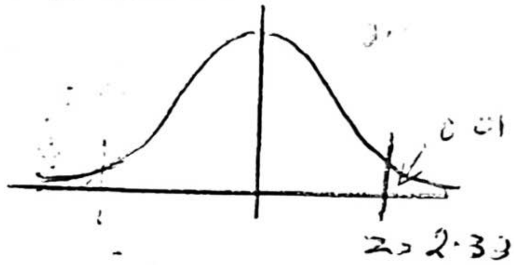
$$= \frac{11.5 - 9.7}{\sqrt{\frac{1.2^2}{61} + \frac{1.6^2}{61}}}$$

$$= \frac{1.8}{\sqrt{0.023606557 + 0.041967213}}$$

$$= \frac{1.8}{\sqrt{0.06557377}}$$

$$= \frac{1.8}{0.256073759}$$

$$= 7.029224729$$



Thus the difference in km per litre is significant and so the null hypothesis is rejected

---

2

$H_0: \mu_g = \mu_b$ and $H_1: \mu_g < \mu_g$

Test Statistic Z

$$= \frac{(\bar{x}_g - \bar{x}_b) - (\mu_g - \mu_b)}{\sqrt{\frac{\sigma_g^2}{n_g} + \frac{\sigma_b^2}{n_b}}}$$

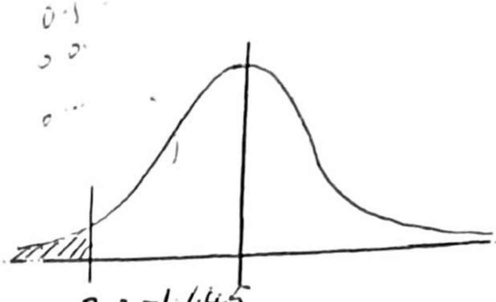
$$= \frac{63.05 - 66.59}{\sqrt{\frac{9.8^2}{108} + \frac{9.6^2}{78}}}$$

$$= \frac{-3.54}{\sqrt{0.889259259 + 1.18153842}}$$

$$= \frac{-3.54}{\sqrt{2.070797721}}$$

$$= \frac{-3.54}{1.439026657}$$

$$= -2.459996124$$



The null hypothesis Ho is rejected as the test statistics lies in the critical region

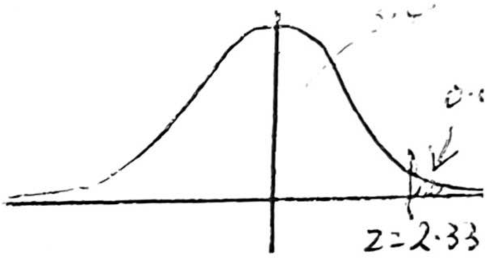
---

3 If the difference between means is denoted by δ such that $\mu_1 - \mu_2 = \delta$, then the null hypothesis $H_0: \delta = 0.05$ and the alternative hypothesis $H_1: \delta < 0.05$

Test Statistic Z

$$= \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{0.135 - 0.082 - 0.05}{\sqrt{\frac{0.004^2}{35} + \frac{0.005^2}{35}}}$$



$$= \frac{0.053 - 0.05}{\sqrt{\frac{0.000016}{35} + \frac{0.000025}{35}}}$$

$$= \frac{0.003}{\sqrt{0.000041/35}}$$

$$= \frac{0.003}{\sqrt{1.171428571 \times 10^{-6}}}$$

$$= \frac{0.003}{1.082325539 \times 10^{-3}}$$

$$= 2.771809304$$

The test statistic lies in the critical

region and so the null hypothesis is rejected

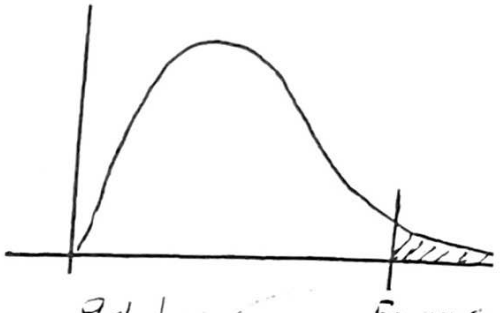
---

4 $\quad H_0: \sigma_2^2 = \sigma_1^2$ and $H_1: \sigma_2^2 > \sigma_1^2$

Test Statistic F

$$= \frac{\text{Larger Variance}}{\text{Smaller Variance}}$$

$$= \frac{9^2}{8^2}$$



$$= 1.265625$$

The test statistic lies in the acceptance region and so the null hypothesis H_0 is accepted. As a result of this, the pooled variance can be used here

Pooled Variance S^2

$$= \frac{(n_1-1)S_1^2 + (n_2-1)S^2}{(n_1+n_2-2)}$$

$$= \frac{9 \times 8^2 + 9 \times 9^2}{(10+10-2)}$$

$$= \frac{9 \times 64 + 9 \times 81}{18}$$

$$= \frac{576 + 729}{18}$$

$$= \frac{1305}{18}$$

$$= 72.5 \text{ i.e. } S^2 = 72.5$$

$H_0: \mu_1 = \mu_2$ and $H_1: \mu_1 \neq \mu_2$

Test Statistic t

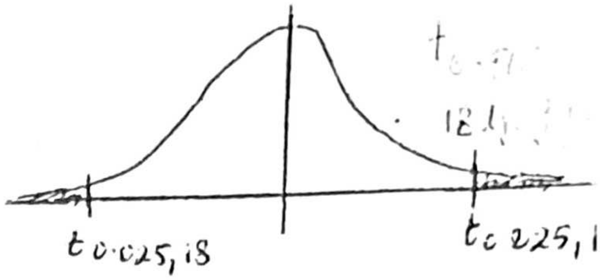
$$= \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S^2/n_1 + S^2/n_2}}$$

$$= \frac{81 - 98}{\sqrt{72.5/10 + 72.5/10}}$$

$$= \frac{-17}{\sqrt{7.25 + 7.25}}$$

$$= \frac{-17}{\sqrt{14.5}}$$

$$= -4.464418717$$



The test statistic lies in the critical region and so the null hypothesis H_0 is rejected

---

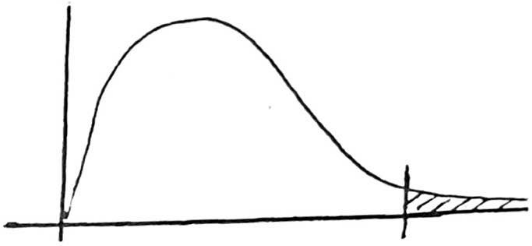
5 $H_0: \sigma_D^2 = \sigma_C^2$ and $H_1: \sigma_D^2 > \sigma_C^2$

Test Statistic F

$$= \frac{\text{Larger Variance}}{\text{Smaller Variance}}$$

$$= \frac{81^2}{57^2}$$

$$= 2.019390582$$



The test statistic lies in the acceptance region as the null hypothesis is accepted. As a result, the variances can be pooled.

Pooled variance s^2

$$= \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{14 \times 57^2 + 14 \times 81^2}{28}$$

$$= \frac{14 \times 3249 + 14 \times 6561}{28}$$

$$= \frac{45486 + 91854}{28}$$

$$= \frac{137340}{28}$$

$= 4905$ ∴ $s^2 = 4905$

$H_0: \mu_D = \mu_C$ and $H_1: \mu_D > \mu_C$.

Test Statistic t

$$= \frac{(\bar{x}_D - \bar{x}_C) - (\mu_D - \mu_C)}{\sqrt{\frac{s^2}{n_D} + \frac{s^2}{n_C}}}$$

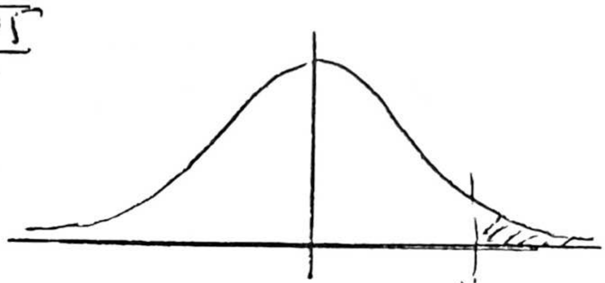
$$= \frac{572 - 515}{\sqrt{\frac{4905}{15} + \frac{4905}{15}}}$$

$$= \frac{57}{\sqrt{327 + 327}}$$

$$= \frac{57}{\sqrt{654}}$$

$$= \frac{57}{25.57342371}$$

$$= 2.228876378$$



$t_{0.05,\,28} = 1.70$

The test statistic lies in the critical region and so the null hypothesis H_0 is rejected

---

i The two sets of data i.e. ratios can't be considered to be independent and so the usual test for the difference between two means can be applied here. The variable that we can use in this case is the difference between the ratios i.e. $d_i = r_{1i} - r_{2i}$

$$\bar{d} = \sum_{i=1}^{12} d_i / 12$$

$$= \frac{85}{12}$$

$$= 7.08\dot{3}$$

see pg 142 my notes
pg 143/144

Sample Variance $s^2 = \frac{1}{11}\left\{\sum_{i=1}^{12} d_i^2 - \frac{\left(\sum_{i=1}^{12} d_i\right)^2}{12}\right\}$

Stock No.	Rate 2009 r_{1i}	Rate 2011 r_{2i}	$d_i = r_{1i} - r_{2i}$	d_i^2
1	40	32	40-32 = 8	64
2	33	23	33-23 = 10	100
3	24	16	24-16 = 8	64
4	21	13	21-13 = 8	64
5	30	23	30-23 = 7	49
6	25	19	25-19 = 6	36
7	19	14	19-14 = 5	25
8	29	21	29-21 = 8	64
9	20	17	20-17 = 3	9
10	35	20	35-20 = 15	225
11	17	13	17-13 = 4	16
12	20	17	20-17 = 3	9
			85	725

$$\text{i.e. } S^2 = \frac{1}{11}\left\{725 - \frac{85^2}{12}\right\}$$

$$= \frac{1}{11}\left\{725 - \frac{7225}{12}\right\}$$

$$= \frac{1}{11}\left\{725 - 602.08\dot{3}\right\}$$

$$= \frac{122.91\dot{6}}{11}$$

$$= 11.17424242$$

$$S = \sqrt{11.17424242}$$

$$= 3.342789616$$

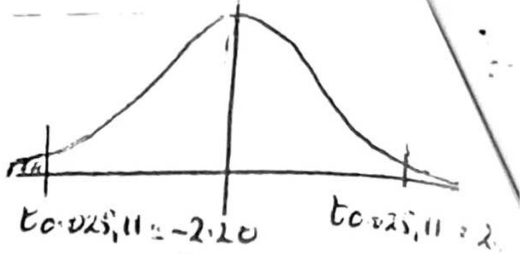
$H_0: \mu_d = 0$ and $H_1: \mu_d \neq 0$

Test Statistic $t = \dfrac{\bar{d} - \mu_d}{S_d/\sqrt{n}}$

$$= \frac{7.08\dot{3} - 0}{3.342789616/\sqrt{12}}$$

$$= \frac{7.083}{0.964980242}$$

$$= 7.340392088$$



The test statistic lies in the critical region and so the null hypothesis H_0 is rejected i.e. there is a change in the price/earning ratios for the two years.

7

Secretary No	Before r_{1i}	After r_{2i}	Diff. $d_i = r_{2i} - r_{1i}$	d_i^2
1	72	75	$75-72=3$	9
2	68	66	$66-68=-2$	4
3	55	60	$60-55=5$	25
4	58	64	$64-58=6$	36
5	52	55	$55-52=3$	9
6	55	57	$57-55=2$	4
7	64	64	$64-64=0$	0
			17	87

$$\bar{d} = \frac{\sum_{i=1}^{7} d_i}{7}$$

$$= \frac{17}{7}$$

$$= 2.428571429$$

$$S_d^2 = \frac{1}{6}\left\{ \sum_{i=1}^{7} d_i^2 - \frac{\left(\sum_{i=1}^{7} d_i\right)^2}{7} \right\}$$

$$= \frac{1}{6}\left\{ 87 - \frac{17^2}{7} \right\}$$

$$= \frac{1}{6}\left\{ 87 - 41.28571429 \right\}$$

$$= \frac{45.71428571}{6}$$

$= 7.619047619$

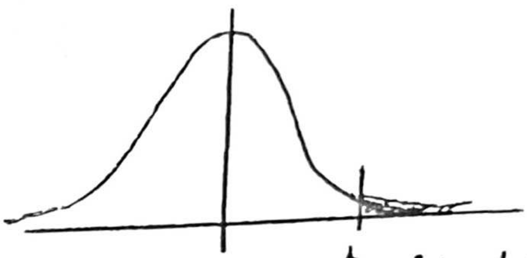
$s = \sqrt{7.619047619}$

$= 2.760262237$

$H_0: \mu_d = 0$ and $H_1: \mu_d = 0$.

Test Statistic t

$= \dfrac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$



$t_{0.05,6} = 1.9$

$= \dfrac{2.428571429 - 0}{2.760262237/\sqrt{7}}$

$= \dfrac{2.428571429}{1.043281062}$

$= 2.32782087$

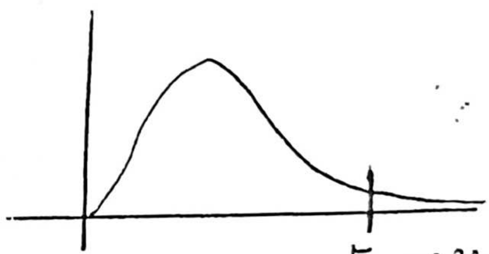
The test statistic lies in the critical region and the null hypothesis H_0 is rejected indicating that mean typing rate has increased.

---

3 $H_0: \sigma_B^2 = \sigma_A^2$ and $H_2: \sigma_B^2 > \sigma_A^2$

Test Statistic F

$= \dfrac{\text{Larger Variance}}{\text{Smaller Variance}}$



$F_{0.05,28,21} = 2$

$= \dfrac{13.20^2}{12.50}$

$= 1.11513$

The test statistic lies in the acceptance region and so the null hypothesis H_0 is accepted. Hence the variances can be pooled.

Pooled Variance s^2

$= \dfrac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{(n_A + n_B - 2)}$

$= \dfrac{21(12.5)^2 + 28(13.2)^2}{22 + 29 - 2}$

$$= \frac{21 \times 156.25 + 28 \times 174.24}{49}$$

$$= \frac{3281.25 + 4878.72}{49}$$

$$= \frac{8159.97}{49}$$

$$= 166.53$$

H_0 $\mu_A = \mu_B$ and $H_1: \mu_A \neq \mu_B$

Test Statistic Z

$$= \frac{(\bar{x}_A - \bar{x}_B) - (\mu_A - \mu_B)}{\sqrt{\frac{S^2}{n_A} + \frac{S^2}{n_B}}}$$

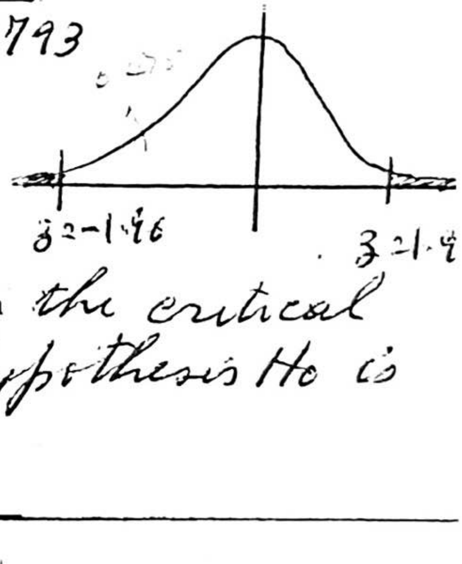
$$= \frac{132 - 123}{\sqrt{\frac{166.53}{22} + \frac{166.53}{29}}}$$

Compare with

$$= \frac{9}{\sqrt{7.569545455 + 5.742413793}}$$

$$= \frac{9}{\sqrt{13.311959243}}$$

$$= 2.466729455$$



The test statistic lies in the critical region and so the null hypothesis Ho is rejected

---

9 $\hat{p}_1 = \frac{412}{5000} \Rightarrow \hat{p}_1 = 0.0824$

$\hat{p}_2 = \frac{411}{4500} \Rightarrow \hat{p}_2 = 0.091\dot{3}$ or $\hat{p}_2 = 0.0913$

Pooled proportion $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$

$= \frac{412+411}{5000+4500}$

$= \frac{823}{9500}$

$= 0.086631578 \Rightarrow \hat{p} = 0.0866$

$H_0: p_2 = p_1$ and $H_1: p_2 > p_1$

Test Statistic z

$= \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{}}}$

$= \frac{(0.0913 - 0.0824)}{\sqrt{\frac{0.0866(1-0.0866)}{5000} + \frac{0.0866(1-0.0866)}{4500}}}$

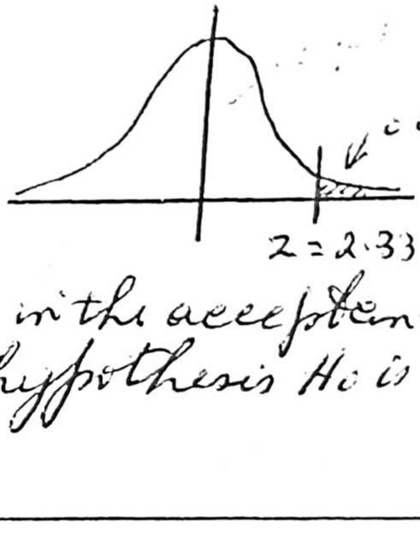
$= \frac{0.0089}{\sqrt{\frac{0.0866 \times 0.9134}{5000} + \frac{0.0866 \times 0.9134}{4500}}}$

$= \frac{0.0089}{\sqrt{1.5820088 \times 10^{-5} + 1.757784556 \times 10^{-5}}}$

$= \frac{0.0089}{\sqrt{3.33979635 \times 10^{-5}}}$

$= \frac{0.0089}{5.779097118 \times 10^{-3}}$

$= 1.540032953$



The test statistic lies in the acceptance region and so the null hypothesis H_0 is accepted.

---

10 $\hat{p}_1 = \frac{74}{200} \Rightarrow \hat{p}_1 = 0.37$

$\hat{p}_2 = \frac{86}{200} \Rightarrow \hat{p}_2 = 0.43$

Pooled proportion $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

$= \dfrac{74 + 86}{200 + 200}$

$= \dfrac{160}{400} \Rightarrow \hat{p} = 0.40$

$H_0: p_1 = p_2$ and $H_1: p_1 \neq p_2$

Test Statistic z

$$= \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n_1} + \dfrac{\hat{p}(1-\hat{p})}{n_2}}}$$

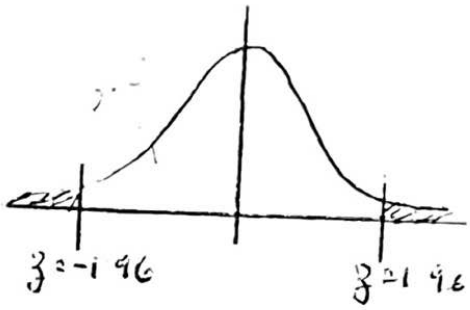
$$= \frac{0.37 - 0.43}{\sqrt{\dfrac{0.4 \times 0.6}{200} + \dfrac{0.4 \times 0.6}{200}}}$$

$$= \frac{-0.06}{\sqrt{0.0012 + 0.0012}}$$

$$= \frac{-0.06}{\sqrt{0.0024}}$$

$$= \frac{-0.06}{0.04898979}$$

$$= -1.22474487$$



The test statistic lies in the acceptance region and so the null hypothesis H_0 is accepted

---

ii

H_0 There is no real relationship between the standard of dress and speed of professional advancement.

H_1 There is a real relationship between the standard of dress and speed of professional advancement.

Observed Data.

	Slow	Average	Fast	Total
Very Well Dressed	32	56	32	120
Well Dressed	28	69	22	119
Poorly Dressed	15	33	13	61
Total	75	158	67	300

Expected Frequency

$30 \times \frac{75}{300} \times \frac{120}{300}$

$= \frac{75 \times 120}{300}$

				Total
Very Well Dressed	$\frac{75\times120}{300}$ $=30$	$\frac{158\times120}{300}$ $=63.2$	$\frac{67\times120}{300}$ $=26.8$	120
Well Dressed	$\frac{75\times119}{300}$ $=29.75$	$\frac{158\times119}{300}$ $=62.67\dot{3}$ $=62.67$	$\frac{67\times119}{300}$ $=26.57\dot{6}$ $=26.58$	119
Poorly Dressed	$\frac{75\times61}{300}$ $=15.25$	$\frac{158\times61}{300}$ $=32.12\dot{6}$ $=32.13$	$\frac{67\times61}{300}$ $=13.62\dot{3}$ $=13.62$	61
Total	75	158	67	300

Test Statistic χ^2

$$= \sum \frac{(O-E)^2}{E}$$

$$= \frac{(32-30)^2}{30} + \frac{(56-63.2)^2}{63.2} + \frac{(32-26.8)^2}{26.8} + \frac{(28-29.75)^2}{29.75}$$

$$+ \frac{(69-62.67)^2}{62.67} + \frac{(22-26.58)^2}{26.58} + \frac{(15-15.25)^2}{15.25} + \frac{(33-32.13)^2}{32.13} +$$

$$\frac{(13-13.62)^2}{13.62}$$

$$= \frac{2^2}{30} + \frac{(-7.2)^2}{63.2} + \frac{(5.2)^2}{26.8} + \frac{(-1.75)^2}{29.75} + \frac{6.33^2}{62.67} + \frac{(-4.58)^2}{26.58} +$$

$$\frac{(-0.25)^2}{15.25} + \frac{(0.87)^2}{32.15} + \frac{(-0.62)^2}{13.62}$$

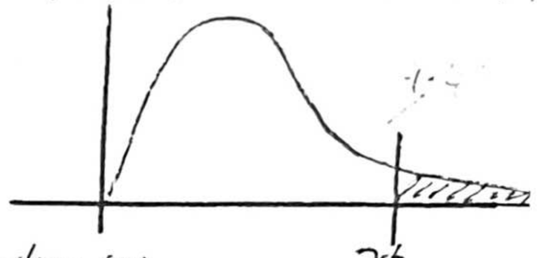
$= 0.1333333 + 0.8202531 + 1.0089552 + 0.10294$
$+ 0.6393633 + 0.7891798 + 0.0040984 + 0.0235574$
0.0282232

$= 3.5499048$

Degrees of freedom

$= (3-1)(3-1)$

$= 4$



degrees of freedom are $(3-1)(3-1)$

$\chi^2_{0.05,4} =$

The test statistics lies in the acceptance region and so the null hypotheses H_0 is accepted

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12

Observed Frequency

	Vendor A	Vendor B	Vendor B	Total
Rejected	12	8	20	40
Not perfect but acceptable	23	12	30	65
Perfect	85	60	110	255
Total	120	80	160	360

H_0: The three vendors ship products of equal quality

H_1: The three vendor do not ship product of equal quality.

Test Statistic χ^2

$$= \sum \frac{(O-E)^2}{E}$$

Expected Frequencies

	VENDOR I	[illegible]	[illegible]	Total
Rejected	$\frac{120\times40}{360}$ $=13.3\dot{3}$ $=13.33$	$\frac{80\times40}{360}$ $=8.8\dot{8}$ $=8.89$	$\frac{160\times40}{360}$ $=17.7\dot{7}$ $=17.78$	40
Not perfect but acceptable	$\frac{120\times65}{360}$ $=21.6\dot{6}$ $=21.67$	$\frac{80\times65}{360}$ $=14.4\dot{4}$ $=14.44$	$\frac{160\times65}{360}$ $=28.8\dot{8}$ $=28.89$	65
Perfect	$\frac{120\times255}{360}$ $=85$	$\frac{80\times255}{360}$ $=56.6\dot{6}$ $=56.67$	$\frac{160\times255}{360}$ $=113.3\dot{3}$ $=113.33$	255
Total	120	80	160	

$$= \frac{(12-13.33)^2}{13.33} + \frac{(8-8.89)^2}{8.89} + \frac{(20-17.78)^2}{17.78} + \frac{(23-21.67)}{21.67}$$

$$+ \frac{(12-14.44)^2}{14.44} + \frac{(30-28.89)^2}{28.89} + \frac{(85-85)^2}{85} + \frac{(60-56.67)^2}{56.67} +$$

$$\frac{(110-113.33)^2}{113.33}$$

$$= \frac{(-1.33)^2}{13.33} + \frac{(-0.89)^2}{8.89} + \frac{(2.22)^2}{17.78} + \frac{(-2.44)^2}{14.44} + \frac{(1.11)^2}{28.89} +$$

$$\frac{0^2}{85} + \frac{(3.33)^2}{56.67} + \frac{(-3.33)^2}{113.33} + \frac{(1.33)^2}{21.67}$$

$$= 0.1327006 + 0.0891001 + 0.2771878 +$$

$$0.4122911 + 0.0426479 + 0 + 0.1956749 + 0.09784$$

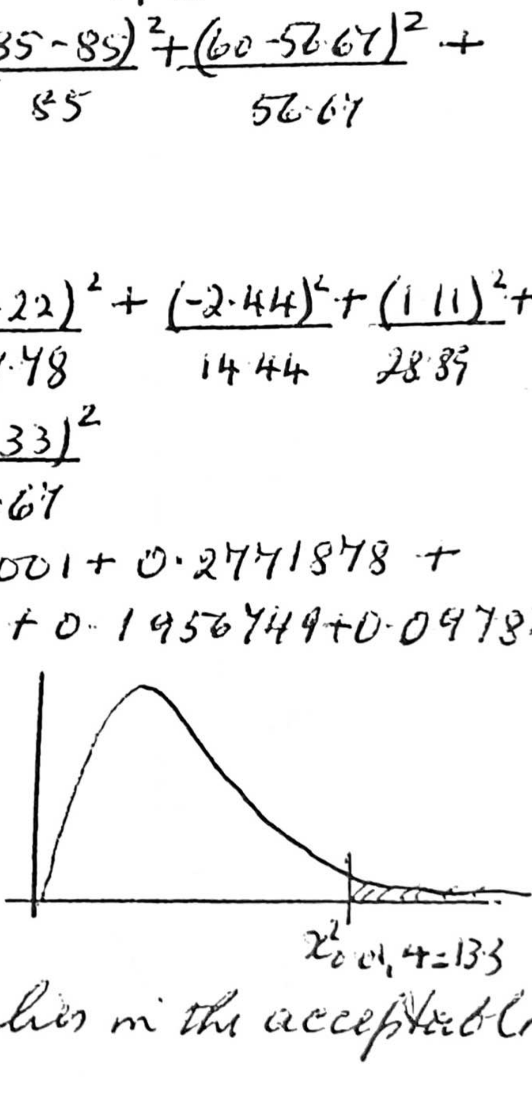
$$+ 0.0816289$$

$$= 1.3290855$$

Degrees of freedom

$= (3-1)(3-1)$ $(r-1)(c-1)$

$= 4$



$\chi^2_{0.01,4} = 13.3$

The test statistics lies in the acceptable region and so the null hypothesis H_0 is accepted

13

Observed Frequency

	Deaf	Blind	Other Handicap	Without Handicap	Total
Above Average	11	3	14	36	64
Average	24	11	39	134	208
Below Average	5	6	7	30	48
Total	40	20	60	200	320

The expected frequencies can be evaluated without firstly defining the null hypothesis H_0 and the alternative hypothesis H_1

Expected Frequency.

	Deaf	Blind	Other Handicap	Without Handicap	Total
Above Average	$\frac{40 \times 64}{320} = 8$	$\frac{20 \times 64}{320} = 4$	$\frac{60 \times 64}{320} = 12$	$\frac{200 \times 64}{320} = 40$	64
Average	$\frac{40 \times 208}{320} = 26$	$\frac{20 \times 208}{320} = 13$	$\frac{60 \times 208}{320} = 39$	$\frac{200 \times 208}{320} = 130$	208
Below Average	$\frac{40 \times 48}{320} = 6$	$\frac{20 \times 48}{320} = 3$	$\frac{60 \times 48}{320} = 9$	$\frac{200 \times 48}{320} = 30$	48
	40	20	60	200	320

The standard χ^2 analysis with 6 degrees of freedom cannot be performed as two of the estimated values in the 'blind' column are less than 5. So the pooling of cells has to be performed before χ^2 analysis can be conducted. The pooling of cells can be performed as suggested in the problem for both the observed and expected data.

Observed Frequency

	With Handicap	Without Handicap	Total
Above Average	28	36	64
Average	74	134	208
Below Average	18	30	48
Total	120	200	320

H_0: The workers' performance is independent of the presence or absence of handicap

H_1: The workers' performance is not independent of the presence or absence of handicap

Expected Frequency

	With Handicap	Without Handicap	Total
Above Average	$\frac{120 \times 64}{320} = 24$	$\frac{200 \times 64}{320} = 40$	64
Average	$\frac{120 \times 208}{320} = 78$	$\frac{200 \times 208}{320} = 130$	208
Below Average	$\frac{120 \times 48}{320} = 18$	$\frac{200 \times 48}{320} = 30$	48
Total	120	200	320

Test Statistic χ^2

$$= \sum \frac{(O - E)^2}{E}$$

$$= \frac{(28-24)^2}{24} + \frac{(36-40)^2}{40} + \frac{(74-78)^2}{78} + \frac{(134-130)^2}{130} + \frac{(18-18)^2}{18} + \frac{(30-30)^2}{30}$$

$$= \frac{(4)^2}{24} + \frac{(-4)^2}{40} + \frac{(-4)^2}{78} + \frac{(4)^2}{130} + \frac{0^2}{18} + \frac{0^2}{18}$$

$= 0.6666667 + 0.4 + 0.2051282 + 0.1230769 + 0$

$= 1.3948718$

Degrees of freedom

$= (3-1)(2-1)$

$= 2$

$\chi^2_{0.05,2} = 5.9$

The test statistic lies in the acceptance region and so the null hypothesis Ho is accepted

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14 Mean

$$= \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{0\times7 + 1\times20 + 2\times12 + 3\times9 + 4\times1 + 5\times1}{7+20+12+9+1+1}$$

$$= \frac{0+20+24+27+4+5}{50}$$

$= \frac{80}{50}$ i.e. Mean $= 1.6$

For binomial distribution mean $= np$ and here $n = 5$ and so mean $= 5p$

$\therefore 5p = 1.6$

$p = \frac{1.6}{5} \Rightarrow p = 0.32$

$\therefore Pr(X = r) = \frac{n!}{r!(n-r)}(0.32)^r(1-0.32)^{n-r}$

$= \frac{n!}{r!(n-r)}(0.32)^r(0.68)^{n-r}$

$Pr(X=0) = \frac{5!}{0!5!}(0.32)^0(0.68)^5$

$= (0.68)^5$

$= 0.1453933568$

$Pr(X=1) = \frac{5!}{1!4!}(0.32)(0.68)^4$

$= 5(0.32)(0.68)^4$

$= 0.3421020016$

$Pr(X=2) = \frac{5!}{2!3!}(0.32)^2(0.68)^3$

$= 10(0.32)^2(0.68)^3$

$= 0.321978368$

$Pr(X=3) = \frac{5!}{3!2!}(0.32)^3(0.68)^2$

$= 10(0.32)^3(0.68)^2$

$= 0.151519232$

$Pr(X=4) = \frac{5!}{4!1!}(0.32)^4(0.68)$

$= 5(0.32)^4(0.68)$

$= 0.035651584$

$Pr(X=5) = (0.32)^5$

$= 3.3554432 \times 10^{-3}$

$= 0.0033554432$

The expected frequencies are evaluated

x	Obs. Freq.	Expected Frequency
0	7	$Pr(X=0)\times 50 = 50\times 0.145393356 = 7.27$
1	20	$Pr(X=1)\times 50 = 50\times 0.342102016 = 17.11$
2	12	$Pr(X=2)\times 50 = 50\times 0.321978368 = 16.10$
3	9	$Pr(X=3)\times 50 = 50\times 0.151519232 = 7.58$
4	$\left.\begin{matrix}1\\1\end{matrix}\right\\} 11$	$Pr(X=4)\times 50 = 50\times 0.035651584 = 1.78$
5		$Pr(X=5)\times 50 = 50\times 0.0033554432 = 0.17$ $\left.\begin{matrix}1.78\\0.17\end{matrix}\right\\}(1.95)$; $\\}\ 9$
	50	50.01

It can be seen that the last two expected frequences are less than 5 and so these can be pooled with the third expected frequency from the bottom.

The modified table is shown below

x	Observed Frequency	Expected Frequency
0	7	7.27
1	20	17.11
2	12	16.10
3,4,5	11	9.53

Test Statistic χ^2

$$= \sum \frac{(O-E)^2}{E}$$

$$= \frac{(7-7.27)^2}{7.27} + \frac{(20-17.11)^2}{17.11} + \frac{(12-16.10)^2}{16.10} + \frac{(11-9.53)^2}{9.53}$$

$$= \frac{(-0.27)^2}{7.27} + \frac{(2.89)^2}{17.11} + \frac{(-4.1)^2}{16.10} + \frac{(1.47)^2}{9.53}$$

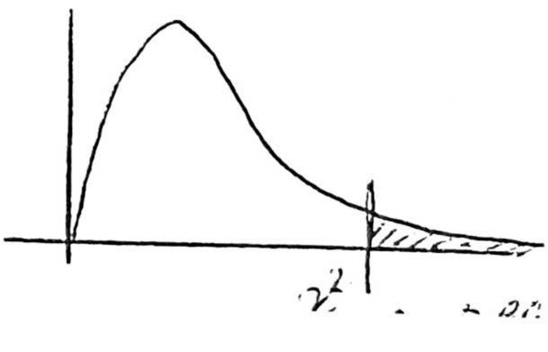
$$= 0.0100275 + 0.4881414 + 1.0440994 + 0.2267471$$

$$= 1.7690154$$

Degrees of freedom

$= 4-1-1$

$= 2$ (subtract one degree of freedom for



The test statistic lies in the acceptance region and so the null hypothesis Ho is accepted

H_0. The data follow binomial distribution

H_1: The data do not follow binomial distribution

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15 Mean.

$$= \frac{0\times5+1\times23+2\times23+3\times25+4\times14+5\times10+6\times0}{5+23+23+25+14+10}$$

$$= \frac{23+46+75+56+50}{100}$$

$$= \frac{250}{100} \quad \text{i.e. Mean} = 2.5$$

$$Pr(X=x) = \frac{e^{-2.5}\,2.5^{x}}{x!}$$

[illegible] another [illegible] the mean [illegible]

H_0: The data fit a Poisson distribution

H_1. The data do not fit a Poisson distribution

x	$Pr(X=x)$	Ob. Freq.	Exp. Freq. = 100 $Pr(X=x)$		
0	0.0820849	5	100×0.0820849	= 8.20849	≃ 8.21
1	0.2052125	23	100×0.2052125	= 20.52125	= 20.52
2	0.2565152	23	100×0.2565152	= 25.65152	= 25.65
3	0.2137630	25	100×0.2137630	= 21.37630	= 21.38
4	0.1336018	14	100×0.1336018	= 13.36018	= 13.36
5	0.0668009	10	100×0.0668009	= 6.68009	= 6.68
6 or more	0.0420212	0	100×0.0420212	= 4.20212	= 4.20
	1.0000000	100			100

$Pr(X \geq 6) = 1 - Pr(X \leq 5)$

$= 1 - 0.9579788 = 1 - [Pr(x=0 \ldots (x=5)]$

$= 0.0420212$ i.e. $Pr(X \geq 6) = 0.0420212$

It can be seen that the last expected frequency is less than 5 and it is pooled with the expected frequency above it. The table is modified as shown below:

x	Obs Freq.	Exp. Freq.
0	5	8.21
1	23	20.52
2	23	25.65
3	25	21.38
4	14	13.36
5 or more	10	10.88

Test Statistic χ^2

$$= \sum \frac{(O-E)^2}{E}$$

$$= \frac{(5-8.21)^2}{8.21} + \frac{(23-20.52)^2}{20.52} + \frac{(23-25.65)^2}{25.65} + \frac{(25-21.38)^2}{21.38} + \frac{(14-13.36)^2}{13.36} + \frac{(10-10.88)^2}{10.88}$$

$$= \frac{(-3.21)^2}{8.21} + \frac{2.48^2}{20.52} + \frac{(-2.65)^2}{25.65} + \frac{(3.62)^2}{21.38} + \frac{(0.64)^2}{13.36} + \frac{(-0.88)^2}{10.88}$$

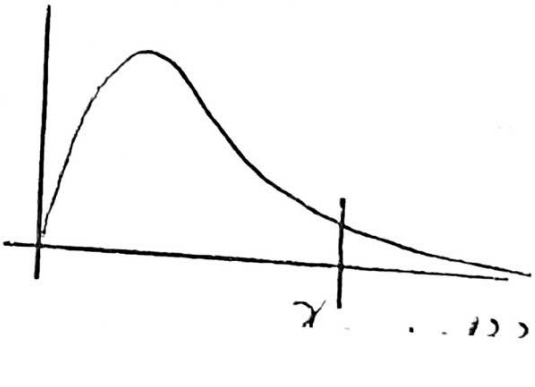
$= 1.255067 + 0.2997271 + 0.2737816 + 0.6129274 + 0.0306586 + 0.0711764$

$= 2.5433381$

Degrees of freedom

$= 6 - 1 - 1$

$= 4$



The test statistic lies in the acceptance region and so the null hypothesis H_0 is accepted.

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16 H_0: The weekly sales follow Poisson distribution with variance 4

H_1: The weekly sales do not follow Poisson distribution with variance 4

$$Pr(X=x) = \frac{e^{-4}4^x}{x!}$$

x	$Pr(X=x)$	Obs. Freq.	Expected Freq $= 50\,Pr(X=x)$
0	0.0183156	3	50X0.0183156 = 0.91578 = 0.92
1	0.0732625	4	50X0.0732625 = 3.663125 = 3.66
2	0.1465251	7	50X0.1465251 = 7.326255 = 7.33
3	0.1953668	11	50X0.1953668 = 9.76834 = 9.77
4	0.1953668	10	50X0.1953668 = 9.76834 = 9.77
5	0.1562934	6	50X0.1562934 = 7.81467 = 7.81
6	0.1041956	6	50X0.1041956 = 5.20978 = 5.21
>6	0.1106741	3	50X0.1106741 = 5.533705 = 5.53
	1.0	50	50

$\left.\begin{matrix}3\\4\\7\end{matrix}\right\}14$ $\qquad$ $\left.\begin{matrix}\left.\begin{matrix}0.92\\3.66\end{matrix}\right\}4.58\\7.33\end{matrix}\right\}11.91$

The first three rows are pooled so that the final entry is greater than 5. The modified table is then obtained

x	Observed Frequency	Expected Frequency
0, 1, 2	14	11.91
3	11	9.77
4	10	9.77
5	6	7.81
6	6	5.21
>6	3	5.53

Test Statistic χ^2

$$= \sum \frac{(O-E)^2}{E}$$

$$= \frac{(14-11.91)^2}{11.91} + \frac{(11-9.77)^2}{9.77} + \frac{(10-9.77)^2}{9.77} + \frac{(6-7.81)^2}{7.81} + \frac{(6-5.21)^2}{5.21} + \frac{(3-5.53)^2}{5.53}$$

$$= \frac{(2.09)^2}{11.91} + \frac{(1.23)^2}{9.77} + \frac{(0.23)^2}{9.77} + \frac{(-1.81)^2}{7.81} + \frac{(0.74)^2}{5.21} + \frac{(-2.53)^2}{5.53}$$

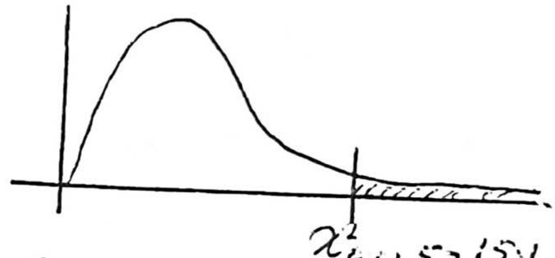
$$= 0.366759 + 0.1548515 + 0.0054145343 + 0.419475 + 0.1197888 + 1.1574864$$

$$= 2.223775234$$

Degrees of freedom

$$= 6-1$$

$$= 5$$



The test statistic lies in the acceptance region and so the null hypothesis H_0 is accepted i.e the weekly sales follow Poisson distribution with variance 4

only one [illegible] the variance [illegible]

[illegible] about the mean

[illegible] only [illegible] total

frequency